Control of Distributed Parameter Systems

July 23-27, 2007

Book of Abstracts

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Foreword

Distributed parameter systems (DPS) is an established area of research in control which can trace its roots back to the sixties. While the general aims are the same as for lumped parameter systems, to adequately describe the distributed nature of the system one needs to use partial differential equation (PDE) models. The modelling issue is in itself nontrivial, especially when there is boundary control action and sensing on the boundary. Controllability and observability concepts are subtle and investigating these for a single PDE example leads to a sophisticated mathematical problem. The action of controlling the system introduces feedback into the PDE model which results in a more complicated mathematical model; the resulting closed-loop system may not be well-posed and this issue has only quite recently become well understood. At this stage, the mathematical machinery for formulating the basic control problems is available (although not so well known), and this has led to a wealth of new system theoretic results for DPS.

If this theory is to be applied, it needs to be tested by numerical simulations of feedback connections of PDE systems, which requires another area of mathematical expertise. Over the past decades considerable experience has been acquired in numerical modelling, simulation and control of DPS for various applications. In particular, much work has been done on the numerical implementation of LQG and miniMax algorithms to various classes of PDE systems. This involves an analytical study of approximations of solutions of operator Riccati or spectral factorization equations, which are reasonably well understood. These approximations lead to a finite-dimensional controller which is designed to stabilize a finite-dimensional approximation of the PDE model. If, however, the controller is to stabilize the original system and not just a simulation of the PDE model, it needs to be robust. Various theories for robust controllers have been proposed, but many open questions remain. More recently, another practical issue, sampled data-control has been addressed. New technology has introduced new control paradigms. In particular, the advent of smart materials for sensors and actuators and micro electro-mechanical actuators and sensors has introduced challenging new modelling and control problems for distributed parameter systems.

Due to the mathematical sophistication of even simply formulated control problems for distributed parameter systems there has been an increasing tendency to specialize on one particular aspect of control. Unfortunately this increasing specialization leads to ignorance of existing expertise in other specializations which could be very appropriate for the problem at hand. The aim of this workshop is perhaps unusual: it is to bring together scientists who are all studying distributed parameter systems, but from different points of view and possessing different types of expertise. In this way, we hope to make scientists aware of new developments in this fast expanding field of research and to promote cross-fertilization of ideas across artificial boundaries. We hope this will open up new directions for future research.

To the best of our knowledge, the last IFAC meeting dedicated to distributed parameter systems was the Fifth IFAC Symposium ”*Control of Distributed Parameter Systems* ”, which was organized by A.El Jai and M. Amouroux, and which took place in Perpignan, France, June 26-29, 1989.

Since then, the DPS community remained of course very active. For example, the 10th International Conference on Analysis and Optimization of Systems was dedicated to the "*State and Frequency Domain Approaches for Infinite-Dimensional Systems* "; it was organized by R.F. Curtain, together with A. Bensoussan and J.L. Lions, in Sophia-Antipolis, France, June 9-12, 1992. In July 1993, H. Logemann organized a "*Workshop on Infinite–Dimensional Control Systems* ", at the University of Bremen, Germany. During the following years, several workshops were organized within the framework of the Human Capital and Mobility European network "Distributed parameter systems: analysis, synthesis and applications": Saariselka - Lapland, Finland, 1994 (Organizer: S. Pohjolainen); Perpignan, France, 1995 (Organizer: A. El Jai); Bath, UK, 1996 (Organizer: H. Logemann). That network was coordinated by S. Townley (University of Exeter, UK) and was active from December 1993 to September 1997.
The workshop CDPS 2007 is the fifth meeting of a series started in 1998 (*Modelling and control of infinite-dimensional systems*, Leeds, UK, September 2-11, 1998 – Organizers: J.R. Partington and S. Townley). The three other meetings were organized in 2001 (*Workshop on Pluralism in Distributed Parameter Systems*, University of Twente, Enschede, The Netherlands, July 2-6, 2001 – Organizers: R.F. Curtain and H. Zwart), 2003 (*International Workshop on Infinite-Dimensional Dynamical Systems*, University of Exeter, UK, July 14-18, 2003 – Organizers: R. Rebarber and S. Townley), and 2005 (*International Workshop on Control of Infinite-Dimensional Systems*, University of Waterloo, Canada, July 25-29, 2005 – Organizers: J. Burns and K. Morris), respectively.

The organizers of CDPS 2007 sincerely hope that there will be a long continuing series of similar meetings dedicated to distributed parameter systems, focused on the same aims, and organized with the renewed support of IFAC.

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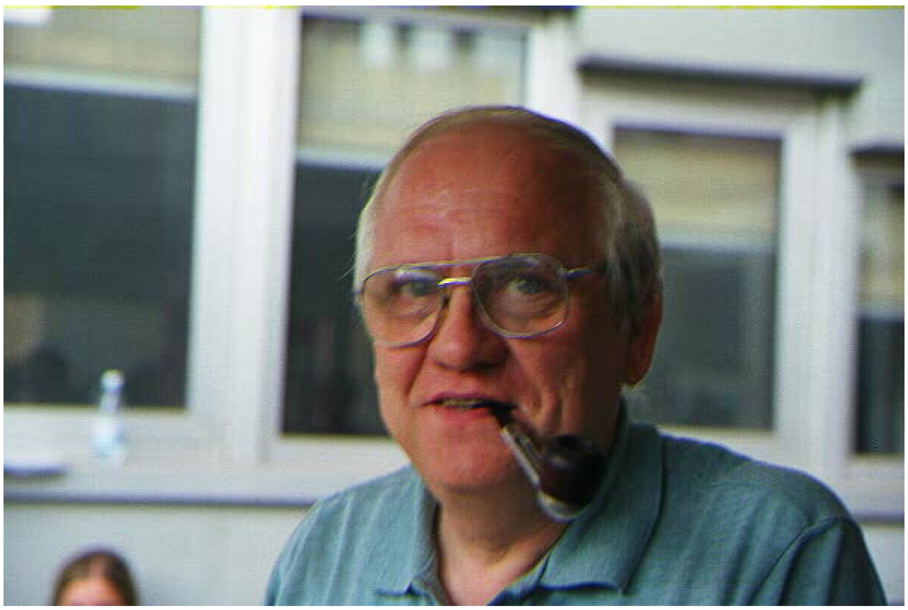
The organizers also wish to thank those of the workshop on *Pluralism in Distributed Parameter Systems*, University of Twente, Enschede, The Netherlands, July 2-6, 2001, who kindly provided them with a good number of source files which were very helpful for editing the web site.

The logo of this workshop is the same than the one of the workshop mentioned above. It has been designed by Hubert van Mastrigt and Hans Zwart. The front cover of the book of abstracts has been designed by Michel Desnoues and Denis Matignon. To these products the normal copyright rules apply.

Contents

This workshop is dedicated to Frank M. CALLIER



Snapshot of Frank enjoying his pipe
during a break at MTNS in Padova, Italy, July 1998

Controller design for DPS

1

Volterra boundary control laws for 1-D parabolic nonlinear PDE's

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Abstract

Boundary control of nonlinear parabolic PDEs is an open problem with applications that include fluids, thermal, chemically-reacting, and plasma systems. We present a stabilizing control design for a broad class of nonlinear parabolic PDEs in 1-D. Our approach is an infinite dimensional extension of the feedback linearization/backstepping approaches for finite dimensional systems employing spatial Volterra series nonlinear operators.

Keywords

Boundary Control, Parabolic Differential Equations, Nonlinear Control

1.1 Introduction

Boundary control of linear parabolic PDEs is a well established subject with extensive literature. On the other hand, boundary control of *nonlinear* parabolic PDEs is still an open problem as far as general classes of systems are concerned.

Our method is a direct infinite dimensional extension of the finite-dimensional feedback linearization/backstepping approaches and employs spatial Volterra series nonlinear operators. We only sketch our method here; a two-part paper [3] has been submitted presenting the method and its properties in full detail, with examples. This result solves open problem 5.1 in the *Unsolved Problems* volume [1].

1.2 Volterra Series

Volterra series represent general solutions for nonlinear equations and are widely studied in the literature [2]. A (spatial) Volterra series is defined as

$$F[u] = \sum_{n=1}^{\infty} \int_0^x \int_0^{\xi_1} \cdots \int_0^{\xi_{n-1}} f_n(x,\xi_1,\ldots,\xi_n) \left(\prod_{j=1}^{n} u(t,\xi_j) \right) d\xi_1 \ldots d\xi_n, \tag{1.1}$$

where f_n is known as the n-th (triangular) kernel of F.

1.3 Outline of the Method

We consider the stabilization problem for the plant

$$
\begin{aligned}
u_t &= u_{xx} + \lambda(x)u + F[u] + uH[u], &(1.2)\\
u_x(0,t) &= qu(0,t), \quad u(1,t) = U(t), &(1.3)
\end{aligned}
$$

where $F[u]$ and $H[u]$ are Volterra series and $U(t)$ the actuation variable. In [3] we show how nonlinear plants found in applications can be written in the form (1.2)–(1.3).

We solve the problem by mapping u into a *target system* w which verifies

$$
\begin{aligned}
w_t &= w_{xx} - cw, &(1.4)\\
w_x(0,t) &= \bar{q}w(0,t), \quad w(1,t) = 0, &(1.5)
\end{aligned}
$$

where $\bar{q} = \max\{0, q\}$. For mapping u into w we use a Volterra transformation

$$
w = u - K[u]. \tag{1.6}
$$

Remark 1.3.1. In [3] we derive the equations that the kernels k_n of K in (1.6) verify. It is a set of *linear* hyperbolic PDEs. For each k_n, we get a PDE evolving on a domain of dimension $n+1$ and with a domain shape in the form of a "hyper-pyramid," $0 \leq \xi_n \leq \xi_{n-1} \ldots \leq \xi_1 \leq x \leq 1$. The equations can be solved recursively, i.e., first for k_1 (which verifies an autonomous equation), then for k_2 (which is coupled with k_1) using the solution for k_1, and so on. We also show in [3] that the Volterra series defined by the k_n's in (1.6) is always convergent and invertible (at least locally).

Once we have the k_n's, the stabilizing control law is determined by (1.6) at $x = 1$

$$
U(t) = \sum_{n=1}^{\infty} \int_0^1 \int_0^{\xi_1} \cdots \int_0^{\xi_{n-1}} k_n(1, \xi_1, \ldots, \xi_n) \left(\prod_{j=1}^{n} u(t, \xi_j) \right) d\xi_1 \ldots d\xi_n. \tag{1.7}
$$

Remark 1.3.2. In [3], using the invertibility properties of K and the exponential stability of (1.4)–(1.5), we show that the origin of the closed-loop system (1.2)–(1.3) with control law (1.7) is exponentially stable in the L^2 and H^1 norms (at least locally). We also illustrate this result with numerical simulations of several examples of interest.

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2

Robust stability of observers

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Keywords

Observer Theory, Strongly Continuous Semigroup, Exponential Stability, Perturbation Theory

In this presentation we consider robust stabilization of a distributed parameter system with an observer [2]. Our aim is to derive conditions under which the compensator stabilizes the system when the system operator used in the compensator differs from the original one.

Let X, U and Y be Hilbert spaces. Consider the system $\Sigma(A, B, C)$ where the operators $A : X \supset \mathcal{D}(A) \to X$, $B \in \mathcal{L}(U, X)$ and $C \in \mathcal{L}(X, Y)$ are such that A generates a C_0-semigroup on X, the pair (A, B) is exponentially stablizable and the pair (A, C) is exponentially detectable. It is well-known that if we choose operators $F \in \mathcal{L}(X, U)$ and $K \in \mathcal{L}(Y, X)$ such that operators $A + BF$ and $A + KC$ generate exponentially stable C_0-semigroups, then the closed-loop system operator A_c generates an exponentially stable C_0-semigroup on $X \times X$.

The replacement of the system operator A with an operator $\widetilde{A}$ in the observer can be seen as a perturbation of the closed-loop system operator A_c. Because of this, we can use theory on the preservation of exponential stablity of C_0-semigroups to derive conditions under which the new system operator $\widetilde{A}_c$ generates an exponentially stable C_0-semigroup.

Because A is an unbounded operator also the perturbation is in general unbounded. We assume that the operator $\widetilde{A}$ is near the original system operator in the sense that $\widetilde{A} - A$ is an A-bounded operator.

We will first derive conditions under which the perturbed system operator $\widetilde{A}_c$ generates a C_0-semigroup on $X \times X$. An application of the perturbation theorem by Miyadera [3] results in conditions involving the C_0-semigroups generated by the operators $A + BF$ and $A + KC$. On the other hand, we can also obtain conditions involving the resolvent operators $R(\lambda, A+BF)$ and $R(\lambda, A+KC)$ by applying the results presented by Kaiser and Weis [1].

Subsequently, we can impose additional conditions under which the C_0-semigroup generated by $\widetilde{A}_c$ is exponentially stable. Because $X \times X$ is a Hilbert space, the C_0-semigroup generated by $\widetilde{A}_c$ is exponentially stable if and only if

$$\sup_{\lambda \in \mathbb{C}^+} \|R(\lambda, \widetilde{A}_c)\| < \infty.$$

We can use this characterization to obtain conditions involving norms of operators $(\widetilde{A} - A)R(\lambda, A + BF)$ and $(\widetilde{A} - A)R(\lambda, A + KC)$. We will also use the theory presented in [4] to derive conditions involving the C_0-semigroups generated by the operators $A + BF$ and $A + KC$.

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3

An H_∞–observer at the boundary of an infinite dimensional system

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Abstract

We design and analyze an H_∞–observer which works at the boundary of an infinite dimensional system with scalar disturbances. The system is a model of a UV disinfection process, which is used in water treatment and food industry.

Keywords

Robust filter design, observers, boundary control theory, H_∞–optimization

3.1 Introduction

In many (control) applications where (bio)chemical reactions and transport phenomena occur, measurement and control actions take place at the boundaries. While a theoretical framework already exist ([1] and references therein), there is little attention to apply this theory in practice, as far as we know.

In [2], the analysis and design of a Luenberger observer for a UV disinfection example is explored. In this paper, we analyze a robust Luenberger-type observer for the same system with boundary inputs and boundary outputs, see [2] for physical background,

$$\frac{\partial x}{\partial t}(\eta,t) = \alpha\frac{\partial^2 x}{\partial \eta^2}(\eta,t) - v\frac{\partial x}{\partial \eta}(\eta,t) - bx(\eta,t), \quad x(\eta,0)=0 \tag{3.1}$$

$$x(0,t) = w_1(t), \quad \frac{\partial x}{\partial \eta}(1,t) = 0, \quad y(t) = x(\eta_1,t) + w_2(t), \tag{3.2}$$

on the interval $\eta \in (0,1)$. Furthermore, α, v, and b are positive constants and corresponding to the diffusion constant, constant flow velocity and micro-organism light susceptibility constant, respectively. The signals $u(t)$, $w_1(t)$, $w_2(t)$ and $y(t)$ represent a scalar input, disturbance (or error) at the inlet boundary ($\eta = 0$), disturbances or errors on the output and a scalar output, respectively.

We design a dynamic Luenberger-type observer,

$$\frac{\partial \hat{x}}{\partial t}(\eta,t) = \alpha \frac{\partial^2 \hat{x}}{\partial \eta^2}(\eta,t) - v\frac{\partial \hat{x}}{\partial \eta}(\eta,t) - b\hat{x}(\eta,t), \quad \hat{x}(\eta,0) = 0 \tag{3.3}$$

$$\hat{x}(0,t) = 0, \quad \frac{\partial \hat{x}}{\partial \eta}(1,t) = K(t) * (y(t) - \hat{y}(t)), \quad y(t) = \hat{x}(\eta_1,t), \tag{3.4}$$

with K to be designed, and $*$ denotes the convolution product. As a consequence, the dynamics for the error $\varepsilon(\eta,t) = x(\eta,t) - \hat{x}(\eta,t)$ is written as

$$\frac{\partial \varepsilon}{\partial t}(\eta,t) = \alpha \frac{\partial^2 \varepsilon}{\partial \eta^2}(\eta,t) - v\frac{\partial \varepsilon}{\partial \eta}(\eta,t) - b\varepsilon(\eta,t), \quad \varepsilon(\eta,0) = 0 \tag{3.5}$$

$$\varepsilon(0,t) = w_1(t), \quad \frac{\partial \varepsilon}{\partial \eta}(1,t) = K(t) * (\varepsilon(\eta_1,t) + w_2(t)). \tag{3.6}$$

Please notice that the correction to possible disturbances w takes place at the boundary.

3.2 H_∞–filter problem

The aim is now to design a K such that the disturbances w_1 and w_2 have hardly any influence on $\varepsilon(1,t)$. This would enable us to predict the value of x at $\eta = 1$ accurately. Since the future of the output cannot be used, we see that K must be causal. We can write this problem as a standard H_∞–filtering problem, i.e. ,

$$\inf_{K \text{ causal}} \sup_{w} \frac{\|\varepsilon(1)\|_2}{\|w\|_2}$$

with $w(t) = \begin{pmatrix} w_1(t) & w_2(t) \end{pmatrix}^\top$.

In [2], we already explored the exponential stability for the error dynamics (3.5)–(3.6) with constant gain. In the talk we shall further outline the procedure of how K is designed for the UV-disinfection example.

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4

Predictive control of distributed parameter systems

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Keywords

Parabolic PDEs, state constraints, input constraints, model predictive control, transport-reaction processes

This talk will present an overview of our recent work on predictive control of various classes of distributed parameter systems. Specifically, we will initially focus on predictive control of linear parabolic PDEs with state and control constraints [5], and design reduced order predictive controller formulations, which upon being feasible, guarantee both stabilization and state constraint satisfaction for the infinite dimensional system. First, the PDE is written as an infinite dimensional system in an appropriate Hilbert space and modal decomposition techniques are used to derive a finite-dimensional system that captures the dominant dynamics of the infinite dimensional system, and express the infinite dimensional state constraints in terms of the finite-dimensional system state constraints. A number of MPC formulations, designed on the basis of different finite-dimensional approximations, will be presented and compared. The closed–loop stability properties of the infinite dimensional system under the low order MPC controller designs are analyzed, and sufficient conditions, which guarantee stabilization and state constraint satisfaction for the infinite dimensional system under the reduced order MPC formulations, are derived. Other formulations are also presented which differed in the way the evolution of the fast eigenmodes are accounted for in the performance objective and state constraints. The impact of these differences on the ability of the predictive controller to enforce closed-loop stability and state constraints satisfaction in the infinite-dimensional system are also analyzed. The MPC formulations are applied, through simulations, to the problem of stabilizing an unstable steady-state of a linearized model of a diffusion-reaction process subject to state and control constraints. Moreover, we extend our approach [4] to deal with nonlinear parabolic PDEs with state and control constraints arising in the context of diffusion-reaction processes and developed computationally-efficient nonlinear predictive control algorithms. Finally, recent results on predictive control of linear parabolic PDEs with boundary control actuation [3], predictive control of linear stochastic parabolic PDEs [8] and predictive control of particulate processes based on population balance models [9] will be discussed.

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Linear systems theory

5

Relation between the growth of $\exp(At)$ and $\big((A+I)(A-I)^{-1}\big)^n$

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Abstract

Assume that A generates a bounded C_0-semigroup on the Hilbert space Z, and define the Cayley transform of A as $A_d := (A+I)(A-I)^{-1}$. We show that there exists a constant $M > 0$ such that $\|(A_d)^n\| \leq M \ln(n+1)$, $n \in \mathbb{N}$.

Keywords

Cayley transform, reciprocal systems, stability.

5.1 Introduction

Consider the abstract differential equation

$$\dot{z}(t) = Az(t), \qquad z(0) = z_0 \tag{5.1}$$

on the Hilbert space Z. A standard way of solving this differential equation is the Crank-Nicolson method. In this method the differential equation (5.1) is replaced by the difference equation

$$z_d(n+1) = (I + \Delta A/2)(I - \Delta A/2)^{-1} z_d(n), \qquad z_d(0) = z_0, \tag{5.2}$$

where Δ is the time step. We denote $(I + \Delta A/2)(I - \Delta A/2)^{-1}$ by A_d.

If Z is finite-dimensional, and thus A is a matrix, then it is easy to show that the solutions of (5.1) are bounded if and only if the solutions of (5.2) are bounded:

$$\sup_{t \geq 0} \|e^{At}\| =: M_c < \infty$$

if and only if

$$\sup_{n\in\mathbb{N}} \|(A_d)^n\| =: M_d < \infty.$$

However, the best estimates for M_d depend on M_c and the dimension of Z, see [2].

If Z is infinite-dimensional, then under the assumption that A and A^{-1} generate a bounded C_0-semigroup e^{At}, and $e^{A^{-1}t}$, respectively, the following estimate has been obtained,

$$M_d = \sup_{n\in N} \|(A_d)^n\| \leq 2e\cdot(M_c^2 + M_{c,-1}^2), \tag{5.3}$$

where $M_c = \sup_{t\geq 0} \|e^{At}\|$ and $M_{c,-1} = \sup_{t\geq 0} \|e^{A^{-1}t}\|$, see [1], [3], and [5]. Note that this estimate is independent of time step Δ.

However, at the moment it is unclear whether the boundedness of the semigroup generated by A implies the existence and the boundedness of the semigroup generated by A^{-1}. So we take another approach to study the behavior of $(A_d)^n$.

5.2 The growth of $(A_d)^n$

In [3] the following result is shown.

Theorem 5.2.1. *Let A generate a bounded C_0-semigroup on the Hilbert space Z, then there exists a constant $M > 0$ such that $\|(A_d)^n\| \leq M\ln(n+1)$ for $n\in\mathbb{N}$.*

The proof of [3] uses estimates on resolvents and contour integrals. We present a proof which is based on techniques from system theory. More precisely, we use Lyapunov equations to obtain the estimate. If the semigroup generated by A is exponentially stable, then for small n's the estimate in Theorem 5.2.1 can be improved. We remark that by posing an extra, nontrivial condition on the resolvent of A, one can prove boundedness of $(A_d)^n$, see [4].

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6

The observer infinite-dimensional Sylvester equation

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Abstract

The paper studies the infinite-dimensional algebraic Sylvester equation as it appears in the designing of an asymptotic observer for a linear infinite-dimensional system. The approach involves the concept of an implemented semigroup, see [1] and [2].

Keywords

Sylvester equation, implemented semigroup, observer design.

6.1 Introduction and problem description

In order to study the infinite-dimensional version of the observer Sylvester equation we introduce the following notation and assumptions

1. The family $(S(-t))_{t\in\mathbb{R}} \subset \mathcal{H}^E$ is a strongly continuous group with generator $(-E, \mathscr{D}(-E))$, where $\mathcal{H}^E := \mathscr{L}(H^E)$.
2. U and Y are Hilbert spaces called the output space and the input space. $B \in \mathscr{L}(U, H^E)$ is the (bounded) input operator. $C \in \mathscr{L}(H_1^E, Y)$ is the (unbounded) output operator.

Under the above assumptions we consider the infinite-dimensional control system

$$\dot{x}(t) = -Ex(t) + Bu(t)\,, \quad x(0) = x_0\,, \tag{6.1a}$$
$$y(t) = Cx(t)\,, \tag{6.1b}$$

where $(x(t))_{t\geq 0}$ is the state trajectory, $(u(t))_{t\geq 0} \subset U$ is the control and $(y(t))_{t\geq 0} \subset Y$ is the output. For the system (6.1) we want to design an *asymptotic state observer*. In order to do that we consider the following infinite-dimensional dynamical system

$$\dot{z}(t) = A_{-1}z(t) + Gy(t) + Hu(t)\,, \quad z(0) = z_0\,, \tag{6.2}$$

where $(z(t))_{t\geq 0}$ is the state trajectory, under the following assumptions:

3. The family $(T(t))_{t\geq 0} \subset \mathcal{H}^A$ is exponentially stable strongly continuous semigroup with generator $(A, \mathscr{D}(A))$, where $\mathcal{H}^A := \mathscr{L}(H^A)$.
4. $G \in \mathscr{L}(Y, H^A_{-1})$ is the output operator. $H \in \mathscr{L}(U, H^A)$ is the input operator.

Under these assumptions we study the following operator equation:

$$A_{-1}Mh + MEh = -GCh\,, \quad h \in H^E_1\,, \tag{6.3}$$

where $M \in \mathcal{H} := \mathcal{L}(H^E, H^A)$ and the equality is understood in H^A_{-1}, and $H = MB$.

6.2 Main results

Since (6.3) has the form of the algebraic Sylvester equation we can now use the results coming from the implemented semigroup theory [1] to see that if $GC \in \mathcal{H}_{-1}$ and $\omega_0(T) + \omega_0(S) < 0$, then (6.3) has a unique solution $M \in \mathcal{H}$. Here the space $\mathcal{H}_{-1}$ plays a crucial role and is defined as the extrapolation space for the implemented semigroup $(\mathcal{U}(t))_{t\geq 0} \subset \mathcal{L}(\mathcal{H})$, where $\mathcal{U}(t)X := T(t)XS(t)$ for $X \in \mathcal{H}$, and $\omega_0(T)$ and $\omega_0(S)$ denote growth bounds of the corresponding semigroups. Since the error $e(t) = z(t) - Mx(t)$ satisfies

$$\|e(t)\|^A = \|T(t)e(0)\|^A \leq C_1 e^{\omega_1 t}\|e(0)\|^A\,, \quad t \geq 0\,, \tag{6.4}$$

where ω_1 is an arbitrary constant satisfying the condition $0 > \omega_1 > \omega_0(T)$, then $\lim_{t\to\infty} \|z(t) - Mx(t)\|^A = 0$. If additionally, the operators A, G and H are such that $M \in \mathcal{H}$ has a bounded inverse $M^{-1} \in \mathscr{L}(H^A, H^E)$, then we have

$$\lim_{t\to\infty} \|M^{-1}z(t) - x(t)\|^E = 0\,. \tag{6.5}$$

This condition means that the system (6.2) is actually an *asymptotic state observer* for the control system (6.1). The rate of convergence (6.5) can be estimated by

$$\|M^{-1}z(t) - x(t)\|^E \leq \|M^{-1}\|_{\mathscr{L}(H^A,H^E)}\|z_0 - Mx_0\|^A C_1 e^{\omega_1 t}\,, \quad t \geq 0\,,$$

and it follows that this convergence may be arbitrary large by a suitable choice of the growth bound $\omega_0(T)$ in the observer (6.2).

6.3 Final comments

The observer design problem based on the Sylvester equation (6.3) can be generalized to a problem where operators E, B and C are given and we are looking for A, G and M such that the equations (6.3) and $H = MB$ hold.

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7

Spectral properties of pseudo-resolvents under structured perturbations

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Abstract

In this talk spectral properties of perturbed closed, densely defined operators on a Banach space are studied.

Keywords

Resolvent linear systems, perturbed closed operators, spectral properties.

7.1 Introduction

The theory of perturbations of unbounded operators is well documented in Kato [3], Pazy [5] and in Engel and Nagel [1]. The results depend crucially on the choice of the class of perturbations. Salamon obtained nice results for structured perturbations of semigroup generators on a Hilbert space in [6] using a feedback approach as used in systems theory. The main aim was to obtain the most general conditions on the triple of unbounded operators A, B, C so that the closed-loop operator $A + BKC$ or some generalization would generate a C_0-semigroup. This was done in [6] and also by Weiss [7] for the class of *well-posed linear systems*. An extension to unbounded perturbations on Banach spaces can be found in Hadd [2]. Our focus in this paper is on spectral properties of the closed-loop operator. As an example we quote a very special case of the result from [6, Lemma 4.4].

Theorem 7.1.1. *Let X, Y, U be Hilbert spaces. Suppose that A is the infinitesimal generator of a C_0-semigroup on X, $B \in \mathcal{L}(U, X), K \in \mathcal{L}(Y, U)$ and $C \in \mathcal{L}(X, Y)$. Then for $\lambda \in \rho(A)$, we have*

$$\lambda \in \rho(A + BKC) \Longleftrightarrow I - KC(\lambda I - A)^{-1}B \text{ is boundedly invertible},$$

In the above, the system has the generating operators A, B, C and the transfer function $G(s) = C(sI - A)^{-1}B$. Under the feedback operator K we obtain the *closed-loop system* $A + BKC, B, C$ with transfer function $G^K(s) = G(s)(I - KG(s))^{-1} = C(sI - A - BKC)^{-1}B$. This framework was generalized to the class of *well-posed linear systems* in [6] and [8]. The main drawback of the above approach is the admissibility assumptions that need to be imposed on B, C which are often difficult to check. In addition, X is assumed to be a Hilbert space. Our main aim in this paper is to obtain analogous perturbation results for closed, densely defined operators A on a Banach space X. Our first step is to obtain structured perturbation results for pseudo-resolvents, see [4].

Definition 7.1.2. Let X be a Banach space and $\Lambda \subset \mathbb{C}$ be a domain. The operator-valued function $\mathfrak{a} : \Lambda \to \mathcal{L}(X)$ is called a *pseudo-resolvent* if it satisfies

$$\mathfrak{a}(\beta) - \mathfrak{a}(\alpha) \quad = \quad (\alpha - \beta)\mathfrak{a}(\beta)\mathfrak{a}(\alpha), \qquad \forall \alpha, \beta \in \Lambda.$$

If there exists a closed, densely defined operator A such that $\mathfrak{a}(\beta) = (\beta I - A)^{-1}$, then the pseudo-resolvent is a resolvent. However, even in this case the closed-loop pseudo-resolvent might not be a resolvent. We give conditions under which this is the case and we generalize Theorem 7.1.1 to closed, densely defined operators on a Banach space.

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8

On the Carleson measure criterion in linear systems theory

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Abstract

In Ho, Russell [3], and Weiss [5], a Carleson measure criterion for admissibility of one-dimensional input elements with respect to for diagonal semigroups is given. In this note we extend their results from the Hilbert space situation $X = \ell_2$ and L^2–admissibility to the more general situation of L^p–admissibility on ℓ_q–spaces. In case of analytic diagonal semigroups we present a new proof showing a link to reciprocal systems in the sense of Curtain [1].

Keywords

Diagonal systems, admissibility, reciprocal systems, Carleson measures

8.1 Introduction

Consider the infinite dimensional linear system described by the differential equation

$$x'(t) + Ax(t) = Bu(t) \qquad x(0) = x_0 \tag{8.1}$$

on a Banach space $X = \ell_q$, $q \in (1, \infty)$. We assume that the injective operator $-A$ is the generator of a bounded diagonal semigroup $S(\cdot)$ acting by $\big(S(t)x\big)_n = \exp(-\lambda_n)x_n$, $n \in \mathbb{N}$ where x_n denotes the n-the component of $x \in X$. Let $B \in B(U, X_{-1})$ where $X_{-1} := \{(\xi_n) : \ (\xi_n/(1 + \lambda_n)) \in X\}$. A solution of (8.1) is necessarily of the form

$$z(t) = S(t)x_0 + \int_0^t S_{-1}(t-s)Bu(s)\, ds$$

Notice that $z(t)$ is a well-defined element of X_{-1} for $t \geq 0$ but generally there is no reason why $z(t)$ should be an element of X. A bounded operator $B \in X_{-1}$ is called *finite-time L^p–admissible* for A, ($p \in [1, \infty]$) if for every $\tau > 0$ there exists a constant $K > 0$ such that

$$\Big\| \int_0^t S_{-1}(t-s)Bu(s)\, ds \Big\|_X \leq K\|u\|_{L^p(0,\tau)} \qquad t \in [0, \tau]$$

If the constant K may be chosen independently of $\tau > 0$, b is called *L^p–admissible* for A. For the special case p=2 there is a large literature on the notion admissibility, we refer to the survey [2] for extended references. In this note we focus on one-dimensional control operators B represented by an element $b \in X_{-1}$. A well-known result of Ho and Russell [3] and Weiss [5] characterises admissibility in case p=q=2 by the Carleson measure property of the associated discrete measure $\mu = \sum_n |b_n|^q \delta_{\lambda_n}$. We present the following extension: Let H^s denote the Hardy space of exponent s on the right half plane. Let $\alpha > 0$. A non-negative measure μ on $\mathbb{C}_+$ is called an α–Carleson measure if the identity, acting $H^{\alpha q} \to L^q(\mu)$, is bounded for one (and thus all) $q \in (1, \infty)$. In case p=q=2 the following result is Ho and Russell [3].

Theorem 8.1.1. *Let $p \in (1, 2]$, $q \in (1, \infty)$ and $\alpha q = p'$ where p' is the dual exponent of p. Then $b = (b_n)$ is an infinite-time L^p–admissible input element for A on $X = \ell_q$ provided that the discrete measure $\mu = \sum_n |b_n|^q \delta_{\lambda_n}$ is an α–Carleson measure.*

In case $\alpha \leq 1$ the condition is in fact necessary and sufficient. For $\alpha = 1$ this is Weiss [5], in case $\alpha \leq 1$ necessity has been considered independently in [4]. For $\alpha > 1$ necessity is still work in progress. A second theorem covers the whole range of values for p and q, but requires analyticity of the semigroup.

Theorem 8.1.2. *Let $p, q \in (1, \infty)$ and $\alpha q = p$. Let $\theta \in (0, \pi/2)$ and let $-A$ be an injective diagonal operator generating an analytic semigroup. Then $b = (b_n) \in X_{-1}$ is an (infinite-time) L^p–admissible input element for A on $X = \ell_q$ provided that the discrete measure μ given by $\mu = \sum_n |\frac{b_n}{\lambda_n}|^q \delta_{\lambda_n^{-1}}$ is an α–Carleson measure.*

Again, in case $\alpha \leq 1$ the criterion is necessary and sufficient, whereas in case $\alpha > 1$ necessity is work in progress. Theorem 8.1.2 may be seen as a result for the reciprocal system

$$z'(t) + A^{-1}z(t) = A^{-1}Bu(t)$$

in the sense of Curtain [1]. This observation allows to give a 'direct' (i.e. involving λ_n instead of λ_n^{-1}) criterion for L^p–admissibility that extends the range of possible values for p, q in Theorem 8.1.1.

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9

Diffusive representation for fractional Laplacian and other non-causal pseudo-differential operators

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Abstract

Diffusive representations are extended to *non-causal* pseudo-differential operators such as $(-\Delta)^\gamma$, for $-\frac{1}{2} < \gamma < \frac{1}{2}$. The idea can be seen as an extension of the Wiener-Hopf factorization and slitting techniques to irrational transfer functions. The interest is twofold: energy inequalities are proved that lead to well-posedness, and stable and efficient numerical schemes are derived, with*out* any hereditary behaviour.

Keywords

Fractional Laplacian, Riesz fractional integro-differentiation, non-causal diffusive representation, Wiener-Hopf factorization

9.1 Introduction

Fractional Laplacian has recently attracted attention in modelling, see e.g. [2, 3], as well as in control theory, see e.g. [4, 6]. Even if the theory of Riesz potentials is not new, and can be accounted for in [7, §. 12. & §. 25.], we find it quite difficult to bridge the gap between these abstract pseudo-differential operators and a concrete way to represent them, in a sense close to realization theory ; even more difficult is the task of deriving stable numerical schemes for such systems.

The aim of this work is to show that elementary first-order systems, either causal or anti-causal, with appropriate aggregation, lead to the representation of *non-causal* pseudo-differential operators, such as : $y = (-\Delta)^{-\beta/2}\,u$, called the Riesz fractional integral of order $0 < \beta < 1$, and $z = (-\Delta)^{+\alpha/2}\,u$, called the Riesz fractional derivative of order $0 < \alpha < 1$. The underlying ideas are those of diffusive representations, see e.g. [8, §. 5] and [5], combined with the Wiener-Hopf techniques, as detailed in e.g. [1, §. 7].

9.2 Non-causal diffusive representations

Following [1, §. 7.1], the stable kernel $h(x) = 2\,e^{-|x|}$ can be decomposed into $h^{\pm}$, with support $\mathbb{R}^{\pm}$; hence the convolution $y = h \star u$ can be seen as the sum of two subsystems, namely $\mp\partial_x \varphi^{\pm}(x) = -1\,\varphi^{\pm}(x) + u(x)$, $\varphi^{\pm}(0) = 0$, and $y = \varphi^{+} + \varphi^{-}$.

We apply this decompostion to $h_\beta(x) = \frac{1}{\Gamma(\beta)}\,|x|^{\beta-1}$, using the diffusive realization of some irrational transfer functions with branchpoints (see [1, §. 7.2]).
Let $\phi^{+}(\lambda, x)$ the *causal* solution of:

$$\begin{array}{rcl} \partial_x \phi^{+}(\lambda,x) & = & -\lambda\,\phi^{+}(\lambda,x) + u(x), \quad \lambda > 0, \quad \phi^{+}(x=0) = 0, \\ y^{+}(x) & = & \int_0^\infty \phi^{+}(\lambda,x)\,\mu_\beta(\lambda)\,d\lambda, \\ z^{+}(x) & = & \int_0^\infty \left[-\lambda\,\phi^{+}(\lambda,x) + u(x)\right]\,\mu_\beta(\lambda)\,d\lambda\,. \end{array} \tag{9.1}$$

Let $\phi^{-}(\lambda, x)$ the *anti-causal* solution of:

$$\begin{array}{rcl} -\partial_x \phi^{-}(\lambda,x) & = & -\lambda\,\phi^{-}(\lambda,x) + u(x), \quad \lambda > 0, \quad \phi^{-}(x=0) = 0, \\ y^{-}(x) & = & \int_0^\infty \phi^{-}(\lambda,x)\,\mu_\beta(\lambda)\,d\lambda, \\ z^{-}(x) & = & \int_0^\infty \left[-\lambda\,\phi^{-}(\lambda,x) + u(x)\right]\,\mu_\beta(\lambda)\,d\lambda\,. \end{array} \tag{9.2}$$

Then, the *standard* output $y := (2\cos 2\beta\pi)^{-1}\,[y^{+} + y^{-}]$ is $y = (-\Delta)^{-\beta/2}u$; whereas the *extended* output $z := (2\cos 2\alpha\pi)^{-1}\,[z^{+} + z^{-}]$ is $z = (-\Delta)^{+\alpha/2}u$, for the particular choices $\alpha = 1 - \beta$ and $\mu_\beta(\lambda) = \frac{\sin\beta\pi}{\pi}\lambda^{-\beta}$.

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Control of systems described by PDE's

10

Motion planning of a reaction-diffusion system arising in combustion and electrophysiology

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Abstract

We consider the approximate controllability of a reaction-diffusion system by controls acting on the boundary. Using a parameterization of the solution involving infinite many integrals of the system, we exhibit a "flat" output for the system. This allows us to prove that the linearized system is approximatively controllable. We also study the motion planning problem and compute the control.

10.1 Introduction

The general class of equations under consideration is a reaction-diffusion system of the following type,

$$\left\{\begin{array}{ll} \Phi_t = \Phi_{xx} + f_1(\Phi,\Psi)\ ,\ \Psi_t = f_2(\Phi,\Psi), & (x,t) \in (0,L)\times(0,T), \\ \Phi_x(0,t) = 0,\ ,\ \Phi_x(L,t) + \Phi(L,t) = G(t), & t \in (0,T), \\ \Phi(x,0) = \Phi_0,\ \Psi(x,0) = \Psi_0, & x \in (0,L). \end{array}\right. \tag{10.1}$$

where, $\Phi = (\phi_1, ..., \phi_n)$ is the vector of diffusing species and $\Psi = (\psi_1, ..., \psi_m)$ is the vector of stored species. The functions f_1 and f_2 are given in $L^\infty(\mathbb{R}^{n+m})^n$ and $L^\infty(\mathbb{R}^{n+m})^m$ respectively and G is the control vector in $L^2(0,T)^n$. This type of system appears in varied domains such as chemistry, electrophysiology (see [2] e.g.), genetics, combustion... Let us consider the example of the NOx-trap catalyst. A NOx trap catalyst can be used to reduce harmful NOx emissions from vehicles that use a combustion mixture with a high amount of oxygen (lean-burn). This is done by storing NOx on the catalyst surface during the time the engine runs lean and subsequently switching the engine to rich operation to reduce the stored NOx. A controller can be used to determine how and at what moment to conduct this switch in order to obtain the best compromise between emission levels and fuel efficiency. After linearization and rescaling, we get the following model (see [1])

$$\begin{cases} \Phi_t = \Phi_{xx} - \Phi + \Psi, \ \ \Psi_t = \Phi - \Psi, & (x,t) \in (0,1) \times (0,+\infty), \\ \Phi_x(0,t) = 0 \ , \ \Phi_x(1,t) + \Phi(1,t) = U(t), & t \in (0,+\infty) \\ \Phi(x,0) = 0, \ \Psi(x,0) = 0, & t \in (0,+\infty). \end{cases} \tag{10.2}$$

where Φ are the gaseous species and Ψ is the proportion of occupied sites. The control $U(t)$ is the concentration of the species Φ at the entrance of the catalytic converter.

10.2 Motion planning of the PDE (10.2)

Let us focus on (10.2), and restrict ourself to the case $\Phi(x,t) \in \mathbb{R}$, and $\Psi(x,t) \in \mathbb{R}$. We may prove that, by letting $S(t) = e^t\Phi(0,t)$, the solutions of (10.2) can be parameterized by S, i.e. we formally compute $\Phi(x,t) = e^{-t}y(x,t)$, $\Psi(x,t) = e^{-t}z(x,t)$, and $U(t) = e^{-t}u(t)$ where

$$y(x,t) = \sum_{n=0}^{\infty} \left[\sum_{k=0}^{n} (-1)^k C_n^k S^{(n-2k)}(t) \right] \frac{x^{2n}}{(2n)!} \ , \tag{10.3}$$

$$z(x,t) = \sum_{n=0}^{\infty} \left[\sum_{k=0}^{n} (-1)^k C_n^k S^{(n-2k-1)}(t) \right] \frac{x^{2n}}{(2n)!} \ , \tag{10.4}$$

$$u(t) = \sum_{n=0}^{\infty} \left[\sum_{k=0}^{n} (-1)^k C_n^k S^{(n-2k)}(t) \right] \frac{1}{(2n)!} + \sum_{n=1}^{\infty} \left[\sum_{k=0}^{n} (-1)^k C_n^k S^{(n-2k)}(t) \right] \frac{1}{(2n-1)!} \ . \tag{10.5}$$

Roughly speaking, this result means that the system (10.2) is “flat-like” since the solutions are parameterized involving infinite many integrals of the system. It is not “flat” in the sense of [3] since we need to integrate S instead of differentiate.

Theorem 10.2.1. *When $S(t)$ is Gevrey of order $\alpha < 2$, the formal solutions* (10.3)-(10.4) *are Gevrey of order α in t and 1 in x and the formal control* (10.5) *is Gevrey of order α.*

We recall that, a smooth function $h : t \in [0,T] \mapsto y(t)$ is Gevrey of order α if there exist M, and R such that for all $m \in \mathbb{N}$, $\sup_{t\in[0,T]} \left| h^{(m)}(t) \right| \leq M \frac{(m!)^\alpha}{R^m}$. We prove the following result of motion planning:

Theorem 10.2.2. *For all $T > 0$, for all Φ_T, Ψ_T in $L^2(0,1)$, we may compute explicitly the control $U(t)$ which approximately steers system* (10.2) *from the initial state $(0,0)$ to the final state (Φ_T, Ψ_T) in time T.*

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11

Control design of a distributed parameter fixed-bed reactor

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Abstract

The Linear-Quadratic optimal control problem is studied for a partial differential equation (PDE) model of a fixed-bed reactor, by using a nonlinear infinite dimensional Hilbert state space description. First the LQ-optimal state feedback operator is computed for the linearized model around a chosen profile along the reactor. A Riccati equation is used for computing the state feedback controller. Then the controller is applied to the nonlinear model, and the resulting closed–loop system dynamical performance is analyzed.

Keywords

Fixed-bed reactors; Infinite-dimensional systems; LQ-optimal control; Asymptotic stability; nonlinear contraction semigroup.

11.1 Model Description

Fixed-bed reactors cover a large class of industrial processes in chemical and biochemical engineering. In most industrial applications of fixed-bed reactors, the reactant wave propagates through the bed with a significantly larger speed than the heat wave because the exchange of heat between the fluid and packing slows the thermal wave down.

The dynamics of fixed-bed reactors are described by nonlinear PDE's derived from mass and energy balances. Here, we consider a fixed-bed reactor with the following elementary chemical reaction (see [3, Section 3.7]): $A \longrightarrow B$. The reaction is endothermic and a jacket is used to heat the reactor. A dynamic model of the process has the form:

$$\rho_p c_{pb} \frac{\partial T}{\partial t} = -\rho_f c_{pf} v_l \frac{\partial T}{\partial z} + (-\Delta H) k_0 C_A \exp(-\frac{E}{RT}) + \frac{U_m}{V_r}(T_j - T) \tag{11.1}$$

$$\bar{\epsilon}\frac{\partial C_A}{\partial t} = -v_l\frac{\partial C_A}{\partial z} - k_0 C_A \exp(-\frac{E}{RT}) \tag{11.2}$$

subject to the initial and boundary conditions:

$$\begin{array}{rclrcl} T(0,t) & = & T_{in}, & T(z,0) & = & T_0(z) \\ C_A(0,t) & = & C_{A,in}, & C_A(z,0) & = & C_{A0}(z) \end{array} \tag{11.3}$$

11.2 Control Design

In [3], a robust controller is designed for this model of a fixed-bed reactor. The controller is synthesized on the basis of a reduced-order slow model, since in this type of reactor the reactant wave propagates through the bed with a significantly larger speed than the heat wave. Here we are interested in the design of an LQ-controller in order to regulate the temperature in the reactor by using T_j as a manipulated input with the understanding that in practice its manipulation is achieved indirectly through manipulation of the jacket inlet flow rate (see [3, Subsection 2.7.4] for more details). Observe that the fixed-bed reactor model can be written as follows:

$$\frac{\partial x}{\partial t} = V\frac{\partial x}{\partial z} + f(x,u) \tag{11.4}$$

The objective of this work is basically two-fold : (a) to extend the Linear-Quadratic problem, studied in [1] for the system (11.4) when the matrix V is diagonal with identical entries, to more general class that includes the fixed-bed reactor model (V diagonal with different entries), (b) to implement this extension to study the Linear-Quadratic problem for the fixed-bed reactor (see [2]).

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12

Scheduling of sensor network for detection of moving intruder

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Abstract

We consider the problem of detecting a moving source in 2D diffusion-advection process, often describing environmental processes, by utilizing a network of sensing devices within the 2D spatial domain. The devices are assumed to have actuating capabilities aimed at containing the moving source by minimizing its effects on the process concentration. In order to increase the source-detecting abilities of the sensor network, these devices measure spatial gradients as opposed to only process concentration. Additionally, the monitoring scheme estimates the process state and at the same time introduces a power management scheme, whereby a subset of the available sensors within the network are kept active over a time interval while the remaining devices are kept dormant. The resulting hybrid infinite dimensional system switches both the actuator, deemed more suitable to contain the source over the duration of a given time interval, and its associated control signal. Extensive simulation studies utilizing at most 16% of the total sensors and 16% of the total actuators used in minimizing the effects of the moving source are presented.

Keywords

Distributed Parameter Systems, Sensor Network, Source Localization.

12.1 Introduction

This work is concerned with the abstract formulation and numerical implementation of a methodology that allows for either the development of an integrated mobile sensor navigation or the fixed-in-space sensor scheduling policy. Additionally, it synthesizes supervisory estimators of diffusion-advection processes having *unknown moving sources* (intruders). It is assumed that the processes under consideration have a sensor network strategically distributed in a spatial domain and it is desired to *activate only a subset* of such a sensor network during a given time interval while the remaining sensors stay dormant. At the same time, as the need arises, to also provide an optimal sensor navigation policy to minimize detection time, and to possibly contain a moving source (intruder).

The process under consideration is taken to be a simplified version of a transport model [4] described by the 2D diffusion-advection partial differential equation

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial \chi}\left(\kappa_{\chi\chi}\frac{\partial c}{\partial \chi}\right) + \frac{\partial}{\partial \psi}\left(\kappa_{\psi\psi}\frac{\partial c}{\partial \psi}\right) - u_\chi \frac{\partial c}{\partial \chi} - u_\psi \frac{\partial c}{\partial \psi} + \mu c + b_1(t,\chi,\psi) + b_2(\chi,\psi)u(t)$$

where $c(t,\chi,\psi)$ denotes the concentration as a function of time t and spatial variables $(\chi,\psi) \in \Omega$. For simplicity, a rectangular domain is assumed with $\Omega = [0, L_\chi] \times [0, L_\psi] \subset \mathbb{R}^2$. For simplicity one assumes that the velocity vector $u = (u_\chi, u_\psi)$ and the (eddy) diffusivities $\kappa_{\chi\chi}(t,\chi,\psi)$, $\kappa_{\psi\psi}(t,\chi,\psi)$ are constant. The spatial function $b_2(\chi,\psi)$ describes the spatial distribution of the actuating devices and $u(t)$ the control signal delivered by these devices to the process. The moving source term with intensity $f(t)$ [1], is located at $\theta_s = (\chi_s, \psi_s)$ and is given by $b_1(t,\chi,\psi) = \delta_\chi(\chi - \chi_s(t))\delta_\psi(\psi - \psi_s(t))\, f(t)$, where $\theta_s(t) = (\chi_s(t), \psi_s(t))$ denotes the point source trajectory within Ω. Following the earlier work in [3], one may assume that partial measurements are available in the form of pointwise information of the concentration $c(t,\chi,\psi)$ at the i^{th} spatial location (χ_i, ψ_i)

$$y_i(t) = c(t,\chi_i,\psi_i) = \int_0^{L_\chi}\int_0^{L_\psi} \delta_\chi(\chi - \chi_i)\delta_\psi(\psi - \psi_i)c(t,\chi,\psi)\, d\chi\, d\psi$$

The above system may be viewed as an evolution equation in a Hilbert space [2]

$$\begin{aligned}\dot{\mathcal{X}}(t) &= \mathcal{A}\mathcal{X}(t) + \mathcal{B}_1(t)f(t) + \mathcal{B}_2 u(t),\\ y_i(t) &= \mathcal{C}_i\mathcal{X}(t), \quad i = 1, 2, \ldots, m,\end{aligned}$$

where $\mathcal{X}(\cdot)$ is the state of the infinite dimensional system and $\mathcal{A}$, $\mathcal{B}_1(t)$, $\mathcal{B}_2$, $\mathcal{C}_i$ are the associated operators. The operator $\mathcal{C}(t)$ incorporates the motion of the sensors, and thus the problem of sensor motion is translated to the time variation of $\mathcal{C}(t)$.

The main objectives of this work are (i) to estimate the process state $c(t,\chi,\psi)$ for all t in a time interval $\mathcal{I}$, $t \in \mathcal{I} \subseteq \mathbb{R}^+$ and all spatial points $(\chi,\psi) \in \Omega$, (ii) to estimate the location $\theta_s(t)$ of the unknown source and (iii) to provide an easily implementable containment policy of the moving source.

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Switched Pritchard-Salamon systems with applications to moving actuators

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Abstract

The objective of this paper is to provide applicable methodologies for optimization problems of a spatially moving (or scanning) actuator within the theoretical framework of switched Pritchard-Salamon systems. Two optimization algorithms are proposed and applied to two relevant examples of moving actuators: a parabolic and a hyperbolic switched system. Some open problems have been also identified. Extensive simulation studies implementing switching control strategies were also performed.

Keywords

Switched Systems, Distributed Parameter Systems, Moving Actuators, Optimal Control.

13.1 Introduction

Many engineering applications consider the use of sensor and actuator networks to provide efficient and effective monitoring and control of processes. In particular, the use of mobile sensors and actuators has been receiving attention as it brings forth an added dimension to the efficient use of sensing and actuating devices as regards to reduction in power consumption, improved performance and efficient monitoring. However, there are gaps between the existing theory and applications. The main objective of this paper is to start filling in one of these gaps. More precisely it is intended to provide applicable methodologies for optimization problems of a spatially moving (or scanning) actuator within the theoretical framework of switched Pritchard-Salamon systems. This is a class of distributed parameter systems that allows for unbounded input and unbounded output operators. Two optimization algorithms are proposed: the first algorithm solves an optimal control problem on a finite-time interval; using the second algorithm one can solve a robust control problem. The algorithms are then applied to two relevant examples of moving actuators: a parabolic and a hyperbolic switched system.

13.2 Problem Formulation

Let $(S_p)_{p\in\mathcal{P}}$, for some index set $\mathcal{P}$, be a family of smooth Pritchard-Salamon systems, of the form $(T(\cdot), B_p, C_p, D)$. We consider the finite-time interval $[t_0, t_f]$, i.e., $t_f < \infty$. To the family $(S_p)_{p\in\mathcal{P}}$, we associate the set

$$\Sigma = \{\sigma \mid \sigma : [t_0, t_f] \to \mathcal{P} \text{ piecewise constant}\}$$

of all possible switches between the given systems.

The *family of switched systems* $((S_p)_{p\in\mathcal{P}}, \Sigma)$ taken under consideration are the hybrid dynamical systems consisting of the family of continuous-time systems $(S_p)_{p\in\mathcal{P}}$ together with all switching rules $\sigma \in \Sigma$, all initial states $x(0) = x_0 \in V$, and all inputs $u \in L_2([t_0, t_f]; U)$ (V and U separable Hilbert spaces).

Assumption 13.2.1. Consider the following assumptions

1. The initial conditions for the state at the beginning of each subinterval are given and they are considered to be the end values of the solution on the preceding time-interval.
2. There are only a finite number $m \geq 2$ of admissible locations for the moving actuator.
3. The time required by the actuating device to traverse from a location to another one is negligible.
4. The choice of the residence time Δt is larger than the minimum dwell time τ_d.

Problem 13.2.2. Given a family of switched systems $((S_p)_{p\in\mathcal{P}}, \Sigma)$ which satisfies Assumptions 13.2.1 and an initial condition $x_0 \in V$, find an optimal control and an optimal switching function that minimize an appropriate cost functional over all possible trajectories of the of $((S_p)_{p\in\mathcal{P}}, \Sigma)$.

Theorem 13.2.3. *Problem 13.2.2 has at least one solution.*

An algorithm for solving Problem 13.2.2 is provided. The algorithm contains six steps structured in two parts.

The result is extended to the robust case where a disturbance is taken into consideration and an associated H_∞ robust control scheme is adapted to the moving actuator case. Numerical results on a parabolic and a hyperbolic system are also presented.

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Control of Distributed Parameter Systems: a tribute to Frank M. Callier

14

The motion planning problem and exponential stabilization of a heavy chain

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Abstract

A model of a heavy chain system with a tip mass is interpreted as an abstract semigroup system on a Hilbert state space. We solve the output motion planning problem using the inverse of the input–output operator. Next, a problem of exponential stabilization is formulated and solved using the colocated stabilizer.

Keywords

infinite–dimensional systems, motion planning problem, exponential stabilization.

14.1 Introduction: A heavy chain system

We consider a heavy chain control system loaded by a lumped mass $m > 0$,

$$\left\{\begin{array}{l} \phi_{tt}(\xi,t) = \left[g\left(\xi+\mu\right)\phi_\xi(\xi,t)\right]_\xi, \quad \xi \in [0,L] \\ \phi_{tt}(0,t) = g\phi_\xi(0,t), \\ \phi(L,t) = u(t), \\ y(t) = \phi(0,t), \end{array}\right\} . \tag{14.1}$$

Here g stands for the acceleration due to gravity, $\mu := \frac{m}{S\rho} = \frac{mL}{M}$ where ρ, L, S and M are, respectively, the density of a chain, its length, are of the cross section and its mass. Let $\phi(\xi,0) = 0$, $\phi_t(\xi,0) = 0$, $u \in \mathrm{C}^2[0,\infty)$ with $u(0) = 0$.

14.2 The semigroup model

$\mathrm{H} := \mathbb{R} \oplus \mathrm{H}^1_L(0,L) \oplus \mathrm{L}^2(0,L)$ where $\mathrm{H}^1_L(0,L) := \left\{\Phi \in \mathrm{H}^1(0,L) :\ \Phi(L) = 0\right\}$ is a closed subspace of the Sobolev space $\mathrm{H}^1(0,L)$. We endow H with the *energetic* scalar product,

$$\left\langle \begin{bmatrix} v \\ \phi \\ \psi \end{bmatrix}, \begin{bmatrix} V \\ \Phi \\ \Psi \end{bmatrix} \right\rangle = \mu v V + \int_0^L g(\xi+\mu)\phi'(\xi)\Phi'(\xi)\mathrm{d}\xi + \int_0^L \psi(\xi)\Psi(\xi)\mathrm{d}\xi$$

Treating, for any fixed $t \geq 0$, the vector $x(t)$,

$$x(t)(\xi) = \begin{bmatrix} v(t) \\ \Phi(\xi,t) \\ \Psi(\xi,t) \end{bmatrix} = \begin{bmatrix} \phi_t(0,t) - \dot{u}(t) \\ \phi(\xi,t) - \mathbf{1}(\xi)u(t) \\ \phi_t(\xi,t) - \mathbf{1}(\xi)\dot{u}(t) \end{bmatrix}, \quad \xi \in [0,L]$$

as an element of H we can rewrite (14.1) into its abstract form

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \mathcal{A}x(t) + d\ddot{u}(t), \\ x(0) & = & d\dot{u}(0) \\ y(t) & = & h^* x(t) + u(t) \end{array} \right\} \overset{X:=x-d\dot{u}}{\Longleftrightarrow} \left\{ \begin{array}{rcl} \dot{X}(t) & = & \mathcal{A}\left[X(t) + d\dot{u}(t)\right], \\ X(0) & = & 0 \\ y(t) & = & h^* X(t) + u(t) \end{array} \right\}, \quad (14.2)$$

$$\mathcal{A}\begin{bmatrix} v \\ \Phi \\ \Psi \end{bmatrix} = \begin{bmatrix} g\Phi'(0) \\ \Psi \\ \left[g(\cdot+\mu)\Phi'(\cdot)\right]' \end{bmatrix}, \ D(\mathcal{A}) = \left\{ \begin{bmatrix} v \\ \Phi \\ \Psi \end{bmatrix} \in \mathrm{H} : \left\{ \begin{array}{c} \Phi \in \mathrm{H}^2(0,L) \\ \Psi \in \mathrm{H}^1_L(0,L) \\ \Psi(0) = v \end{array} \right\} \right\},$$

$$d = \begin{bmatrix} -1 \\ \mathbf{0} \\ -\mathbf{1} \end{bmatrix} \in \mathrm{H} \setminus D(\mathcal{A}), \qquad h = \frac{1}{g}\begin{bmatrix} 0 \\ -\ln(\cdot+\mu) + \ln(L+\mu) \\ \mathbf{0} \end{bmatrix} \in D(\mathcal{A}).$$

Theorem 14.2.1. *$\mathcal{A}$ has a countable spectrum consisting entirely of purely imaginary single eigenvalues $\lambda_{\pm n} \sim \pm j\frac{n\pi}{\beta-\alpha}$, $n \in \mathbb{N}$, and a set of corresponding eigenvectors which is an orthonormal basis of H. $\mathcal{A}$ generates a unitary group $\{S(t)\}_{t\in\mathbb{R}}$ on H.*

14.3 The output motion planning problem

We wish to find a control u which gives rise to a given, sufficiently smooth, output trajectory. If $y \in \mathrm{C}^4[0,\infty)$ with $\operatorname{supp} y = [\beta-\alpha,\infty)$ is a given (planned) output trajectory then

$$\ddot{u}(t) = \frac{1}{2\pi}\int_0^t \det\begin{bmatrix} p(\tau) & q(\tau) \\ y^{(4)}(t-\tau-\beta+\alpha) & y^{(4)}(t-\tau+\beta-\alpha) \end{bmatrix}\mathrm{d}\tau\ , \quad (14.3)$$

with $u(0) = \dot{u}(0) = \ddot{u}(0) = 0$, where $p, q \in \mathrm{L}^1(0,T)$ for any $T > 0$,

$$p(t) = \int_0^t \frac{\mathbb{1}(2\beta-\tau)}{\sqrt{2\beta\tau-\tau^2}}\ \frac{2\,(\alpha+t-\tau)^2-\alpha^2}{\sqrt{(t-\tau)^2+2\alpha(t-\tau)}}\,\mathrm{d}\tau \sim 2\pi(\alpha+t-\beta)\ ,$$

$$q(t) = \int_0^t \frac{\mathbb{1}(2\alpha-\tau)}{\sqrt{(t-\tau)^2+2\beta(t-\tau)}}\ \frac{(\tau-\alpha)^2}{\sqrt{2\alpha\tau-\tau^2}}\,\mathrm{d}\tau - \int_0^t \frac{\mathbb{1}(2\alpha-\tau)\sqrt{2\alpha\tau-\tau^2}}{\sqrt{(t-\tau)^2+2\beta(t-\tau)}}\,\mathrm{d}\tau\ ,$$

where $\mathbb{1}(t)$ denotes Heaviside step function.

14.4 Exponential stabilization of the chain at a final position

If a final position of the chain is reached then a problem is to stabilize this position. To solve this problem we use a negative, physically realizable, feedback control law of the *colocated*–type:

$$\dot{u}(t) = -kd^{\#}X = -kd^{\#}x, \quad k > 0 \ ,$$
$$d^{\#}X = g(L+\mu)\Phi'(L), \quad D(d^{\#}) = \{X \in \mathrm{H} : \Phi' \text{ is continuous at } \theta = L\}$$

Theorem 14.4.1. *The closed–loop system operator $\mathcal{A}_c$,*

$$\mathcal{A}_c X := \mathcal{A}\left[X - kdd^{\#}X\right], \qquad D(\mathcal{A}_c) = \{X \in D(d^{\#}) : \ X - kdd^{\#}X \in D(\mathcal{A})\}$$

generates on H a C_0–semigroup of contractions which is **EXS** .

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A historical journey through the internal stabilization problem

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Abstract

The purpose of this talk is to give a historical but personal journey through the internal stabilization problem. We study the evolution of the mathematical formulation of this concept and its characterizations from the seventies to the present day. In particular, we explain how the different mathematical formulations allow one to parametrize all the stabilizing controllers of an internally stabilizable plant. Finally, we focus on the important contributions of F. M. Callier on the internal stabilization problem of classes of infinite-dimensional systems.

Keywords

Internal stabilization problem, parametrization of all stabilizing controllers, doubly coprime factorizations, infinite-dimensional linear systems, fractional representation approach, fractional ideals, lattices, algebraic analysis.

Recognizing when a real plant can be stabilized by means of a feedback law is one of the oldest issues in automatic control. This problem, developed for clear practical reasons, was recently abstracted within the mathematical language in order to be studied on its own and generalized to larger and larger classes of systems, slowly passing from the engineer world to the mathematical one. With a very few concepts such as controllability, observability and robustness, the concept of stabilizability is one of the main interesting cross-fertilizations between very practical engineering problems and mathematics. The evolution of this new mathematical concept should attract more attention from science historians and researchers as we shall show.

We want to take the opportunity of the celebration of F. M. Callier's scientific career who, with C. A. Desoer, G. Zames, M. Vidyasagar, B. A. Francis and others, has brought significant contributions to the study of this concept particularly for infinite-dimensional linear systems ([3, 4, 5, 7, 9, 21, 26]), to give a historical but personal journey through the internal stabilization problem. We are convinced that there is a lot to learn from the historical study of this central concept. Reading directly the papers where this concept was created, developed and used (see, e.g., [8, 10, 13, 16, 22, 27] and the references therein) is a source of enlightenment, bringing a new light on the evolutions developed since and the comings and goings between different approaches. See [2, 24] for some historical accounts.

We study the evolution of the mathematical formulation of the concept of internal stabilizability and its characterizations from the seventies to the present day. We explain how the different mathematical formulations allowed one to parametrize all the stabilizing controllers of the corresponding plant. We emphasize on the fractional representation approach developed by M. Vidyasagar, C. A. Desoer, F. M. Callier, B. A. Francis and others based on the existence of doubly coprime factorizations of the transfer matrices ([6, 10, 15, 22, 23]) and on a mainly forgotten approach developed by G. Zames and B. A. Francis based on the particular transfer matrix $Q = C\,(I - P\,C)^{-1}$ ([13, 27]). See also [1, 2, 11, 12] for the second one. In particular, we focus on the significant contributions of F. M. Callier on the internal stabilization problem of infinite-dimensional linear systems (see, e.g., [3, 4, 5, 7]).

We explain how the use of modern algebraic techniques (fractional ideals, lattices, modules) allows us to show that the approach developed by G. Zames and B. A. Francis ([13, 27]) supersedes the classical fractional representation approach ([6, 10, 15, 22, 23]). Within this lattice approach ([18, 19]), we give general necessary and sufficient conditions for internal stabilizability and for the existence of (weakly) doubly coprime factorizations of irrational transfer matrices. Moreover, we give a general parametrization of all stabilizing controllers of an internally stabilizable plant which reduces to the classical Youla-Kučera parametrization ([10, 14, 25]) when the plant admits a doubly coprime factorization ([18, 20]). The knowledge of only one stabilizing controller is required to get this new parametrization.

Finally, we explain why the lattice approach was historically developed in algebra by Kummer, Dedekind and their followers at the end of the nineteen century for solving conditions similar to the ones obtained from the characterization of internal stabilizability (and from Lamé's famous mistake on Fermat's last theorem). Hence, the use of this mathematical theory was very natural and allowed us to develop our results before realizing that the main ideas could be traced back to the pioneering work of G. Zames and B. A. Francis ([13, 27]). These ideas could not have been completely realized for general classes of systems as the authors did not know the fractional ideal and lattice approaches. Therefore, this shows that old approaches can sometimes be still fruitful when the corresponding mathematical techniques are mature even if, as it was unfortunately our case, we had to preliminary rediscover them before investigating the past literature! The moral of this story advocates for the better knowledge of the historical development of our field and explains the topic of this talk, hoping closing the loop!

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Approximate tracking for stable infinite-dimensional systems using sampled-data tuning regulators[1]

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Keywords

Disturbance rejection, frequency-domain methods, infinite-dimensional systems, input-output methods, internal model principle, low-gain control, sampled-data control, tracking.

Consider the sampled-data feedback system shown in the figure below.

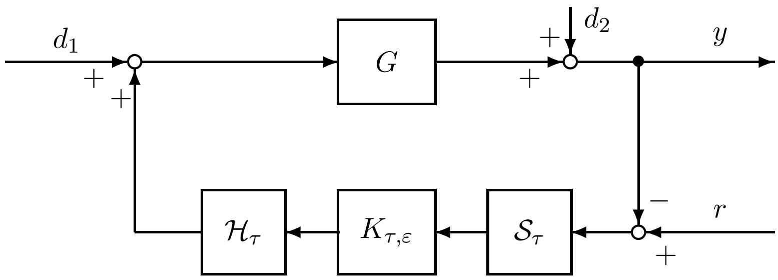

Figure: Sampled-data feedback system

We assume that

- G is a convolution operator with kernel μ, where μ is a $\mathbb{C}^{p\times m}$-valued Borel measure on $\mathbb{R}_+$ such that $\int_{\mathbb{R}_+} e^{\alpha t}|\mu|(dt) < \infty$ for some $\alpha > 0$, where $|\mu|$ denotes the total variation of μ;

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- the reference signal r is of the form

$$r(t) = \sum_{j=1}^{N} e^{\xi_j t} r_j$$

  and the disturbance signals $d_1 : \mathbb{R}_+ \to \mathbb{C}^m$ and $d_2 : \mathbb{R}_+ \to \mathbb{C}^p$ satisfy

$$\lim_{t\to\infty} (d_1(t) - \sum_{j=1}^{N} e^{\xi_j t} d_{1j}) = 0\,, \quad \lim_{t\to\infty} (d_2(t) - \sum_{j=1}^{N} e^{\xi_j t} d_{2j}) = 0\,,$$

  where $\xi_j \in i\mathbb{R}$, $r_j \in \mathbb{C}^p$, $d_{1j} \in \mathbb{C}^m$ and $d_{2j} \in \mathbb{C}^p$ for $j = 1, \ldots, N$;

- $\mathcal{H}_\tau$ and $\mathcal{S}_\tau$ denote the (zero-order) hold and (ideal) sampling operators, respectively, where $\tau > 0$ is the sampling period;

- the discrete-time controller $K_{\tau,\varepsilon}$ is such that its transfer function $\mathbf{K}_{\tau,\varepsilon}$ is of the form

$$\mathbf{K}_{\tau,\varepsilon}(z) = \varepsilon \sum_{j=1}^{N} \frac{K_j}{z - e^{\xi_j \tau}}\,,$$

  where $K_j \in \mathbb{C}^{m\times p}$, $j = 1, \ldots, N$.

Under the assumption that

$$\text{spectrum}(\mathbf{G}(\xi_j)K_j) \subset \{s \in \mathbb{C} : \operatorname{Re} s > 0\}\,, \quad j = 1, \ldots, N\,,$$

where $\mathbf{G}$ denotes the transfer function of G, it is shown that

- there exists $\tau^* > 0$ such that, for every sampling period $\tau \in (0, \tau^*)$, there exists $\varepsilon_\tau > 0$ such that, for all $\varepsilon \in (0, \varepsilon_\tau)$, the sampled-date feedback system is L^∞-stable;

- for every $\delta > 0$ there exists $\tau_\delta > 0$ such that, for every sampling period $\tau \in (0, \tau_\delta)$, there exists $\varepsilon_\tau > 0$ such that, for every $\varepsilon \in (0, \varepsilon_\tau)$,

$$\limsup_{t\to\infty} \|y(t) - r(t)\| \le \delta\,.$$

This result provides a sampled-data counterpart to the continuous-time low-gain regulator results proved in [1, 2].

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17

Problems of robust regulation in infinite-dimensional spaces

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Abstract

In this paper problems of robust regulation for infinite-dimensional systems are discussed. A simple presentation for robust regulators and a derivation of the Internal Model Principle will be given for infinite-dimensional systems with infinite-dimensional exosystems.

Keywords

Robust regulation, Internal Model Principle, Strong stabilization, Infinite-dimensional systems, Distributed parameter systems.

17.1 Introduction

One of the cornerstones of the classical automatic control theory for finite-dimensional linear systems is the Internal Model Principle (IMP) due to Francis and Wonham, and Davison. Roughly stated, this principle asserts that any error feedback controller which achieves closed loop stability also achieves robust (i.e. structurally stable) output regulation (i.e. asymptotic tracking/rejection of a class of exosystem-generated signals) if and only if the controller incorporates a suitably reduplicated model of the dynamic structure of the exogenous reference/disturbance signals which the controller is required to track/reject.

In this paper we discuss the state space generalization of the Internal Model Principle for infinite-dimensional systems with infinite-dimensional signal generators, which generate reference and disturbance signals of the form

$$\sum_{n=-\infty}^{\infty} a_n e^{i\omega_n t}, \qquad \omega_n \in \mathbb{R}, \quad (a_n)_{n\in\mathbb{Z}} \in \ell^1. \tag{17.1}$$

The presentation is based on the concept of the steady state behavior of the closed-loop system with inputs of the form (17.1). This approach leads us naturally to an infinite-dimensional Sylvester equation and a constrained infinite-dimensional Sylvester equation, which adds a constraint for regulation. It is shown that feedback structure enables robustness, as the regulation equation is contained in the Sylvester's equation and as the system reaches its steady state this equation is automatically satisfied. Finally it will be shown that if the controller contains a sufficiently rich internal model of the exosystem, then the Sylvester equation implies robust regulation.

Due to the fact that the signal generator is infinite-dimensional, the closed-loop system cannot be exponentially stabilized. Instead strong stabilization must be used.

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A tribute to Frank M. Callier

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Abstract

The aim of this brief text is, on behalf of all the people attending CDPS 2007 and of all the members of the ”System and Control Community”, and especially those of the ”Distributed Parameter Systems community”, to thank Frank Maria Callier for all he did and is still doing for our scientific community, in Belgium and all over the world. We all know his modesty and humility; nevertheless we are sincerely convinced that he deserves such a tribute.

If you ask me to describe Frank in one word, I would say: researcher. This is the word which can describe him best. In addition, Frank is a wise man; this assertion is very well illustrated by one of his favorite mottoes: ” Beter een vogel in de hand dan tien in de lucht ” . He has always focused his research activities on fundamental questions in system and control theory, without studying too many different problems at the same time, and always with the same simple goal: understanding in depth.

When speaking to Frank, you quickly notice that there is a word, which comes quite often out of his mouth: Berkeley. He spent several years at the University of California, in Berkeley, where he got his Ph.D., in engineering and computer science in 1972. Charles Desoer was his thesis advisor. At that time he was already involved in the study of Distributed Parameter Systems (DPS): he extended the well-known Nyquist stability criterion to such systems.
In 1979, he received an Honorable Mention Paper Award of the IEEE Control Systems Society (Institution of Electrical and Electronics Engineers, New York), jointly with Wan Chan and Charles A. Desoer, [3].

One of the most outstanding contributions of Frank, if not the most outstanding and famous one, is certainly the invention and the development of what is commonly called the Callier-Desoer algebra of transfer functions for DPS (1978), [4], [5] [6]. This is by now the standard class of transfer functions which people usually work with in DPS theory, [13], [10]. This class can be seen as a subclass of H-infinity, which encompasses all DPS of interest in applications. At least, as far as I know, nothing better has been found so far.

You may also know that Frank is one of the first contributors to the factorization approach (he prefers to use the word: fraction) for feedback control system synthesis, [15]. He established the parameterization of all stabilizing controllers for DPS in a paper published in the *Annales de la Société Scientifique de Bruxelles*, [6], at the beginning of the 80's, some short time before the publication of the famous (general paper) by Desoer, Liu, Murray and Sacks.

Frank is also an expert in spectral factorization and Riccati equations. He published several fundamental papers on these topics, in particular with Jacques Willems (on the convergence of the Riccati differential equation), [12], and with myself, [11], [16]. Frank is really a fan of spectral factorization. One of his most important contributions is certainly the paper on the spectral factorization problem of polynomial matrices, where he played one of his favorite games: the massage of the point at infinity, [1].

He also wrote two books, both jointly with Charles Desoer: a research monograph on the polynomial approach to multivariable feedback systems, [7], and a textbook on linear system theory, [8]. These books may appear to be hard to read, when reading them superficially. However, if you look at the details, you will easily observe that they are extremely carefully written and they contain numerous fundamental and solid concepts and results. These books have been cited a numerous amount of times in the literature and the second one has been used as a textbook reference for several university courses, especially in the US.

Frank has also been elected fellow of the IEEE for his contributions to multivariable feedback system theory. This was made known all over our country, by articles published in Belgian newspapers.

Frank did not directly supervise many doctoral thesis. However he had very important and strong influences on a lot of young people, notably as an active member of a good number of doctoral thesis committees.
His outstanding work as reviewer of numerous papers, and as Associate Editor of *Systems and Control Letters*, *Automatica*, and *IEEE Transactions on Automatic Control* , and as Associate Editor at Large of the latter, was and is still highly appreciated by all his colleagues.

He is very exacting, for others, but first for himself. When working on a specific research topic and when writing papers with him, you will quickly observe that he is hard to please and that he does not like at all to rush for publication. Instead he prefers to take the time to analyze again and again all the facets of the same question in depth, he prefers to write and rewrite a part of a paper (or a whole paper), until he reaches a final result which pleases him and which he believes will be not too far from the final result after review.
When he reads a paper, he really does it in detail. He does even more: he rewrites the whole paper for himself, even by rediscovering the proofs contained in the paper, without reading them in advance. He is really impressive.

Frank likes teaching very much. As a professor, he has educated numerous students in

mathematics, notably by giving fundamental courses of mathematical analysis, viz. topology, and measure and integration (including Fourier transform), introductory courses of optimal control and optimal feedback systems, and a master course on semigroup theory, [2].
He is known by several of us and by his students in Namur, as being a citation specialist. To be more precise, he likes to state, especially when he is teaching, some short sentences, which translates in a very pictorial way his goal or his feelings at a particular specific time of a course. This happens for example when he introduces a new concept or notation, or when he explains a proof of a theorem.

Frank has often told me that he does not want to be seen as a piece of museum. Of course he is not: he has been and is still active, as it can be seen on his personal home page *http://perso.fundp.ac.be/∼fcallier/Callier05.pdf*. This is also confirmed by his recent contributions, [9], [14], where one can observe once more his extreme care in writing scientific papers, and his excellent abilities as engineer and applied mathematician.

I wish to address to him again my sincere thanks and those of all my colleagues, for all he has done, and also for what he is still presently doing. Good luck to him and his family for all his future projects and activities.

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Neutral systems

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Stabilization of fractional delay systems of neutral type with single delay

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Abstract

We give here a complete characterization of H_∞-stability of a class of fractional delay systems of neutral type with single delay. In a particular case, the set of all H_∞-stabilizing controllers is given.

Keywords

fractional system, delay system, H_∞ stability, neutral system

19.1 Statement of the problem

We consider here fractional delay systems with transfer function of the form

$$G(s) = \frac{r(s)}{p(s) + q(s)\, e^{-sh}}$$

where $h > 0$ and p, q, r are real polynomials in the variable s^μ for $0 < \mu < 1$. The condition that the system be of neutral type is that $\deg p = \deg q$. Also we take $\deg p \geq \deg r$ in order to deal with proper systems.

We first adapt the Walton and Marshall technique in order to be able to decide on the presence of poles of G in the closed right half-plane.

Then we derive necessary and sufficient conditions in terms of $\deg p$ and $\deg r$ to characterize H_∞-stability of G.

Those results are used in order to find H_∞-controllers for G. In the particular case $\deg p = \deg q = 1$, we show that G is stabilizable by a fractional PI controller, that is a controller with transfer function $K(s) = k_p + \dfrac{k_i}{s^\mu}$.

From this particular controller we can get a parametrization of the set of all stabilizing controllers.

In the case where G has all its poles of large modulus asymptotic to a vertical line strictly in the left half-plane, we can give closed-form solutions involving a free parameter in H_∞. This method has the merit of not relying on the solutions of transcendental equations as is the case when determining Bézout factors given coprime factors.

Theorem 19.1.1. *Let* $G(s) = \dfrac{1}{as^\mu + b + (cs^\mu + d)e^{-sh}}$ *with* $a, b, c, d \in \mathbb{R}$, $a > 0$, $c \neq 0$.

Suppose that $|a| > |c|$*; then the set of all* H_∞*-stabilizing controllers is given by* $\dfrac{V + MQ}{U - NQ}$, *where*

$$\begin{aligned} N(s) &= \frac{1}{s^\mu + 1}, \qquad M(s) = \frac{(as^\mu + b) + (cs^\mu + d)e^{-sh}}{s^\mu + 1}, \\ U(s) &= \frac{s^\mu (s^\mu + 1)}{((as^\mu + b) + (cs^\mu + d)e^{-sh})s^\mu + k_p s^\mu + k_i}, \\ V(s) &= \frac{(s^\mu + 1)(k_i + k_p s^\mu)}{((as^\mu + b) + (cs^\mu + d)e^{-sh})s^\mu + k_p s^\mu + k_i}, \end{aligned}$$

Q *is a free parameter in* H_∞ *and* $k_i > 0$ *and* k_p *satisfy*

$$(a(b + k_p) - cd) \cos \frac{\pi}{2}\mu > 0,$$

$$(b + k_p)^2 + 2ak_i \cos \pi\mu - d^2 > 0,$$

and

$$k_i(b + k_p) \cos \frac{\pi}{2}\mu > 0.$$

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Stability and computation of roots in delayed systems of neutral type

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Abstract

In this paper we give methods for checking the location of poles of neutral systems with multiple delays. These are of use in determining exponential stability and H_∞-stability in the single delay case.

Keywords

SOS tools, delay system, neutral system

20.1 Statement of the problem

We consider the problem of stability of systems with characteristic equations of the form

$$G(s) = G_1(s) + \sum_{i=2}^{n} G_i(s) e^{-\tau_i s}, \qquad \text{where} \qquad G_i(s) = \sum_{j=1}^{m} a_{ij} s^j,$$

for $a_{ij} \in \mathbb{R}$ and $\tau_i \geq 0$. Suppose we are given the values of a_{ij} and would like to determine whether the system is stable, either in the exponential or H_∞ sense, for a given set of values of τ. In this paper, we give results which allow us to answer two distinct questions.

1. **Delay-Independent Stability:** Is G exponentially stable for $\tau_i \geq 0$?
2. **Delay-Dependent Stability:** For given h_i, is G H_∞-stable for $\tau_i \in [0, h_i]$?

Our work gives results which allow us to reformulate the problem in terms of semialgebraic sets. We then use Positivstellensatz results to express the problem as convex optimization over sum-of-squares polynomials. We use semidefinite programming to solve the optimization numerically. We use the version of the Positivstellensatz given by Stengle [3].

Theorem 20.1.1 (Stengle). *The following are equivalent*

1. $\left\{x : \begin{array}{ll} p_i(x) \geq 0 & i = 1, \ldots, k \\ q_j(x) = 0 & j = 1, \ldots, m \end{array}\right\} = \emptyset$
2. *There exist* $t_i \in \mathbb{R}[x]$, $s_i, r_{ij}, \ldots \in \Sigma_s$ *such that*

$$-1 = \sum_{i=1}^{m} q_i t_i + s_0 + \sum_{i=1}^{k} s_i p_i + \sum_{\substack{i,j=1 \\ i \neq j}}^{k} r_{ij} p_i p_j + \cdots$$

Here $\mathbb{R}[x]$ denotes the set of real-valued polynomials in variables x and Σ_s denotes the subset of $\mathbb{R}[x]$ which admit a sum-of-squares representation. For a given degree bound, the conditions associated with Stengle's positivstellensatz can be represented by a semidefinite program since for any $s_i \in \Sigma_s$, there exists a matrix $Q \geq 0$ such that $s(x) = Z(x)^T Q Z(x)$, where Z is a vector of monomials in x. The connection between semidefinite programming and sum-of-squares was first made by Parillo [1].

Delay-Independant Stability In this case, we use the following very simple stability condition.

Proposition 20.1.2. *Suppose that for some* $\epsilon > 0$, $\{s : G_1(s) + \sum_{i=2}^{n} G_i(s) z_i = 0, \operatorname{Re} s \geq -\epsilon, \|z_i\|^2 \leq 1 + \epsilon\} = \emptyset$. *Then* G *is exponentially stable for any* $\tau_i \geq 0$.

Using the Positivstellensatz, we construct a semidefinite program which checks the conditions of the Lemma. This is illustrated using a number of numerical examples.

Robust Delay-Dependent Stability In this case, we use an approach first considered by Zhang et al. [4]. This method was based on two principles; 1) The location of the rightmost root of G is a continuous function of the values of the delay τ and 2) A robust version of the Padé approximation can be used to enclose the function $e^{-j\omega}$ on the imaginary axis.

For neutral systems, principle 1 holds for $\tau > 0$, but not necessarily at $\tau = 0$. Therefore, we must check that new roots appear in the left half-plane for infinitessimal τ and in the particular case of a single delay, we have a condition [2] which characterizes this. In the case of multiple commensurate delays, we use a more conservative condition given in terms of the $a_{i,n}$.

Once the above conditions have been satisfied, we can apply robust Padé approximants in the spirit of [4]. We can then use the Positivstellensatz to construct semidefinite programming conditions. This approach is illustrated with numerical examples.

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What can regular linear systems do for neutral equations?

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Abstract

Let $A : \mathcal{D}(A) \subset X \to X$ be the generator of a strongly continuous semigroup on a Banach space X, and let the operators $D, L : W^{1,p}([-r, 0], X) \to X$ be linear and bounded. Denote

$$\mathcal{X} = X \times L^p([-r, 0], X) \quad \text{with norm} \quad \|\left(\begin{smallmatrix} z \\ \varphi \end{smallmatrix}\right)\| = \|z\| + \|\varphi\|_p.$$

Consider the linear operator $\mathcal{A}_D : \mathcal{D}(\mathcal{A}_D) \subset \mathcal{X} \to \mathcal{X}$ defined by

$$\mathcal{A}_D := \begin{pmatrix} A & L \\ 0 & \frac{\partial}{\partial\theta} \end{pmatrix},$$
$$\mathcal{D}(\mathcal{A}_D) := \left\{ \left(\begin{smallmatrix} z \\ \varphi \end{smallmatrix}\right) \in \mathcal{D}(A) \times W^{1,p}([-r, 0], X) : z = D\varphi \right\}.$$

We note that the operator $\mathcal{A}_D$ is closely related to neutral equations with difference operator D and delay operator L.

We consider the following:

Problem 1 *Find general conditions on D and L for which $\mathcal{A}_D$ generates a strongly continuous semigroup on $\mathcal{X}$.*

Generally, in neutral equations, the works consider atomic operator D, that is $D\varphi = \varphi(0) - K\varphi$, where K is nonatomic at zero (e.g. [1, Sect. 6], [4, Chap. 9]). Here we give a new semigroup approach to Problem 1 mainly based on closed-loop systems and a Perturbation theorem of Staffans–Weiss (see [5, Chap. 7], [6]). We shall also see how this approach allows us to prove that the semigroup generated by $\mathcal{A}_D$ is eventually compact whenever the semigroup generated by A is immediately compact. This will serves to use well-known

criterion for the stabilization of distributed linear system to introduce general conditions for the feedback stabilization of neutral equations.

But, it is of much importance to solve Problem 1 in the case of D nonatomic at zero, and it is natural to expect such a situation in control problems such as aeroelastic systems. The none atomicity of D makes many difficulties for directly applying the concept of closed–loop systems. However, we shall present an approach which allows us to use Staffans–Weiss perturbation theorem in an indirect way ([2]). We note that the operators D and L should be issued as observation operators of regular linear systems governed by the left shift semigroup on $L^p([-r,0],X)$ (see [3]).

Finally, we consider the singular neutral reaction–diffusion equation

$$\begin{aligned}
&\frac{d}{dt}\Big(\int_{-r}^{0} c|s|^{-\frac{1}{2}}u(t+s,x)\,ds\Big) = \sum_{k=1}^{n}\frac{\partial^2}{\partial x_k^2}\Big(\int_{-r}^{0} c|s|^{-\frac{1}{2}}u(t+s,x_k)\,ds\Big)+\\
&\qquad\qquad a\int_{-r}^{0} u(t+s,x)\,d\varpi(s)+f(t,x),\ \ x\in\Omega,\ t\geq 0,\\
&\int_{-r}^{0} c|s|^{-\frac{1}{2}}u(t+s,x)\,ds=0, \qquad\qquad x\in\partial\Omega,\ \ t\geq 0,\\
&\qquad\qquad x(s,x)=\varphi(s,x), \qquad\qquad a.e.\ (s,x)\in[-r,0]\times\Omega,
\end{aligned} \tag{21.1}$$

where $c,a>0$ are some constants, $x=(x_1,\cdots,x_n)$, $\Omega\subset\mathbb{R}^n$ a bounded open set with boundary $\partial\Omega$ and $\varpi:[-1,0]\to[0,1]$ is a function of bounded variation (one can consider ϖ as the Cantor function, which is singular with total variation 1).

We shall see that the equation (21.1) is well–posed only on weighted spaces.

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On controllability and stabilizability of linear neutral type systems

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Abstract

Linear systems of neutral type are considered using the infinite dimensional approach. Conditions for exact controllability and regular asymptotic stabilizability are given. The main tools are the moment problem approach and the existence of a Riesz basis of invariant subspaces.

Keywords

Neutral type systems, Riesz basis, exact controllability, stabilizability.

22.1 Statement of the problem

In this paper we consider the problem of controllability and stabilizability for a general class of neutral systems with distributed delays given by the equation

$$\dot{z}(t)-A_{-1}\dot{z}(t-1) = Lz_t(\cdot) = \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\mathrm{d}\theta+\int_{-1}^{0} A_3(\theta)z(t+\theta)\mathrm{d}\theta+Bu(t), \quad (22.1)$$

where A_{-1} is a constant $n \times n$-matrix, A_2, A_3 are $n \times n$, L_2 valued matrices. We consider the operator model of the neutral type system (22.1) in the product space $M_2 = \mathbb{C}^n \times L_2(-1, 0; \mathbb{C}^n)$, so (22.1) can be reformulated as

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t),\ x(0) = \begin{pmatrix} y \\ z(\cdot) \end{pmatrix},\ \mathcal{A} = \begin{pmatrix} 0 & L \\ 0 & \frac{\mathrm{d}}{\mathrm{d}\theta} \end{pmatrix},\ \mathcal{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \quad (22.2)$$

with $\mathcal{D}(\mathcal{A}) = \{(y, z(\cdot)) \in M_2 : z \in H^1([-1, 0]; \mathbb{C}), y = z(0) - A_{-1}z(-1)\}$, and $\mathcal{A}$ is the generator of a C_0-semigroup. The reachability set $\mathcal{R}_T$ is such that $\mathcal{R}_T \subset \mathcal{D}(\mathcal{A})$ for all $T > 0$, with $u(\cdot) \in L_2$, the solution of (22.2) being in $\mathcal{D}(\mathcal{A})$.

Theorem 22.1.1. *The system (22.2) is exactly null-controllable, i.e.* $\mathcal{R}_T = \mathcal{D}(\mathcal{A})$, *iff the pair* (A_{-1}, B) *is controllable and* $\operatorname{rank}\begin{pmatrix}\Delta_{\mathcal{A}}(\lambda) & B\end{pmatrix} = n$ *for all* $\lambda \in \mathbb{C}$, *where*

$$\Delta_{\mathcal{A}}(\lambda) = \lambda I - \lambda \mathrm{e}^{-\lambda} A_{-1} - \lambda \int_{-1}^{0} \mathrm{e}^{\lambda s} A_2(s) \mathrm{d}s - \int_{-1}^{0} \mathrm{e}^{\lambda s} A_3(s) \mathrm{d}s,$$

If these conditions hold then the system is controllable at any time $T > n_1$, *where* n_1 *is the controllability index of the pair* (A_{-1}, B). *It is not controllable at* $T \leq n_1$.

The main tools of the analysis is the moment problem approach and the theory of basis of exponential families. We construct a special Riesz basis using the existence of a Riesz basis of invariant subspaces [5] and describe the controllability problem via a moment problem in order to get the time of controllability. See [3] for the monovariable and discrete delay case, via a different approach, and [4] for a preliminary result.

The same Riesz basis of subspaces allows to characterize the problem of asymptotic stabilizability by a regular feedback law. From the operator point of view, the regular feedback law

$$u = \mathcal{F}x = \int_{-1}^{0} F_2(\theta)\dot{z}(t+\theta)\mathrm{d}t + \int_{-1}^{0} F_3(\theta) z(t+\theta)\mathrm{d}t, \tag{22.3}$$

where $F_2, F_3 \in L_2(-1, 0; \mathbb{C}^{n\times n})$ means a perturbation of $\mathcal{A}$ by the operator $\mathcal{BF}$ which is relatively $\mathcal{A}$-bounded and verifies $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A} + \mathcal{BF})$. Such a perturbation does not mean, in general, that $\mathcal{A} + \mathcal{BF}$ is the infinitesimal generator of a C_0-semigroup. However, in our case, this fact is verified directly since after the feedback we get also a neutral type system like (22.1) with $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A} + \mathcal{BF})$. This feedback law is essentially different from that which use the term $F\dot{x}(t-1)$ (cf. for example [2]) and for which $\mathcal{D}(\mathcal{A}) \neq \mathcal{D}(\mathcal{A} + \mathcal{BF})$. Our main result is

Theorem 22.1.2. (Rabah, Sklyar & Rezounenko) *Under the assumptions: the eigenvalues of the matrix* A_{-1} *satisfy* $|\mu| \leq 1$, *the eigenvalues* $\mu_j, |\mu_j| = 1$ *are simple, the system (22.1) is regularly asymptotically stabilizable if* $\operatorname{rank}\begin{pmatrix}\Delta_{\mathcal{A}}(\lambda) & B\end{pmatrix} = n$ *for all* $\lambda : \operatorname{Re}\lambda \geq 0$, *and* $\operatorname{rank}\begin{pmatrix}\mu I - A_{-1} & B\end{pmatrix} = n$ *for all* $\mu : |\mu| = 1$.

In the case when A_{-1} has at least one eigenvalue $|\mu| = 1$ with a nontrivial Jordan chain, the system can *not* be stabilized by a control of the form (39.1). The same if $\sigma(A_{-1}) \not\subset \{\mu : |\mu| \leq 1\}$. This follows from the fact that any control of the form (39.1) leaves the system in the same form and then it remains unstable [5].

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23

Coprime factorization for irrational functions

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Abstract

We consider coprime factorizations for irrational functions with a special emphasis on state space formulas.

Keywords

Coprime factorizations.

23.1 Introduction

Coprime factorizations of transfer functions have been studies for some 30 years now. One of the main applications to control theory is the Youla-Jabr-Bongiorno-Kucera parametrization of all stabilizing controllers for a given plant which is given in terms of a coprime factor and the corresponding Bezout factors, but there are many more important applications of the concept of coprime factorization in control theory.

There is a strong connection between coprime factorization and linear quadratic regulator theory which can be used to calculate the coprime factorization and the Bezout factors in terms of a state space realization of the transfer function (see [6],[8] for the rational case). In this talk we will focus on this state space approach.

The finite-dimensional state-space solution readily generalizes to the case of exponentially stabilizable and detectable systems with bounded finite rank input and output operators [5, Chapter 7]. What happens if one droppes the exponential stabilizability and detectability assumption was studied in [4] (for positive real strongly stabilizable systems) and [2] (for strongly stabilizable systems). The assumptions on the input and output operator were generalized in [3] (while keeping the exponential stabilizability and detectability condition). In [9] the notion of joint stabilizability-detectability (which is weaker than exponential stabilizability and detectability) was introduced and shown to be equivalent to the existence of coprime factorizations (for the very general class of well-posed linear systems).

In [1] it was shown that the finite cost condition for the system itself and its dual is equivalent to the existence of coprime factorizations. This assumption is a priori weaker than the earlier joint stabilizability-detectability assumption and can be checked in practical PDE examples (in contrast with the joint stabilizability-detectability assumption). The equivalence was shown for the class of distributional control systems (which includes the class of well-posed linear systems as a subclass). It is this last mentioned work [1] that we will mainly discuss in this talk.

Finally we wish to note that under the finite cost condition for the system alone (not also for the dual system) existence of weakly coprime factorizations has been proven [7]. For some purposes weakly coprime factorizations are good enough, but for other purposes the earlier mentioned (strongly) coprime factorizations are essential.

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Energy methods

24

A class of passive time-varying well-posed linear systems

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Abstract

Starting from a time-invariant dissipative system, we construct a class of time-varying systems by introducing a time-dependent inner product on the state space and modifying some of the generating operators. This class of linear systems is motivated by physical examples such as the electromagnetic field around a moving object.

Keywords

Well-posed linear system, operator semigroup, linear time-varying system, scattering passive system, Maxwell equations.

24.1 Introduction and main result

Various classes of time-varying linear systems with inputs and outputs have been introduced in the papers [1], [2], [3], and others. The most general definition is the one in [3] which mimicks the concept of a (time-invariant) well-posed linear system from Weiss [5]. Unfortunately, for such systems, there is no complete representation theory available (unlike for time-invariant well-posed systems). In fact, already for time-varying systems without inputs and outputs the relevant theory (developed by Kato) is much less complete than the theory of strongly continuous semigroups in the time-invariant case. It is difficult to verify that a given system of linear equations defines a time-varying well-posed system, and for this reason it is also difficult to construct non-trivial examples of such systems. The difficulties arise when we have unbounded control or observation operators and the system is not of parabolic type.

In this paper we introduce a class of time-varying well-posed linear systems. Each such system is constructed using a dissipative (or scattering passive) time-invariant system and a family of time-dependent inner products on the state space.

Let Σ^i be a scattering passive time-invariant system in the sense of [4] (where such systems were called 'dissipative') with generating operators $(A, B, \overline{C}, D)$, state space X, input space U and output space Y (which are Hilbert spaces). Let $P : \mathbb{R}_+ \to \mathcal{L}(X)$ be a twice strongly continuously differentiable function such that $P(t) = P(t)^* > 0$ and $P(t)^{-1}$ is bounded for every $t \geq 0$. We introduce a new system Σ, informally defined by the equations

$$\dot{x}(t) = AP(t)x(t) + Bu(t), \tag{24.1}$$

$$y(t) = \overline{C}P(t)x(t) + Du(t). \tag{24.2}$$

Here the domain of $AP(t)$ may heavily depend on $t \geq 0$, but it can be seen that the extrapolation space of $AP(t)$ is isomorphic to the extrapolation space X_{-1} of A. Recall that X_{-1} is the completion of X w.r.t. $\|(\omega I - A)^{-1}x\|$, for some $\omega \in \rho(A)$, and that $B : U \to X_{-1}$ and $\overline{C} : D(A) + (\omega I - A^{-1})BU \to Y$ are continuous, cf. [4].

Theorem 24.1.1. *Under the above assumptions, let $\tau \geq 0$ and $(x(\tau), u) \in X \times H^1_{loc}([\tau, \infty), U)$ with $Ax(\tau) + Bu(\tau) \in X$. Then* (24.1) *has a unique solution $x \in C^1([\tau, \infty), X)$ and* (24.2) *defines the output function $y \in H^1_{loc}([\tau, \infty), Y)$. The operators $AP(t)$ generate an evolution family $T(t, \tau)$, $t \geq \tau \geq 0$, which has a continuous extension to X_{-1}, and it holds $x(t) = T(t, \tau)x(\tau) + \int_\tau^t T(t, r)Bu(r)\, dr$ for every $t \geq \tau \geq 0$. The balance inequality*

$$\tfrac{d}{dt} \langle P(t)x(t), x(t) \rangle \leq \|u(t)\|^2 - \|y(t)\|^2 + \langle \dot{P}(t)x(t), x(t) \rangle \tag{24.3}$$

holds for every $t \geq \tau \geq 0$. If the original time-invariant system Σ^i is energy preserving, then we have equality in (24.3)*. The map $(x(\tau), u|[\tau, t]) \mapsto (x(t), y|[\tau, t])$ defines a well–posed time–varying system Σ in the sense of [3].*

There is a version of this result if $P(\cdot)$ is just C^1. Our theorem can be applied to Maxwell equations with energy preserving boundary control and observation and time-varying permittivity and permeability. In this example, one can think of a mechanism which changes, say, the permittivity by moving an iron bar inside the domain without changing the total energy of the system. A preliminary analysis based on Theorem 24.1.1 indicates that one can establish (local in time) well-posedness of the resulting energy preserving, time-invariant, quasilinear, coupled system.

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25

Lyapunov control of a particle in a finite quantum potential well

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Abstract

A Lyapunov-based approach for the trajectory generation of a Schrödinger equation is proposed. For the case of a quantum particle in a 3-dimensional finite potential well with an arbitrary shape the convergence is precisely analyzed.

Keywords

Schrödinger equation, Quantum systems, Stabilization, Dispersive estimates.

25.1 Introduction

The control of an infinite dimensional quantum system, in general, poses much harder problems than the finite dimensional case. Concerning the controllability problem, very few results are available [1, 3]. Concerning the trajectory generation problem, still less results are available. In particular, the few available controllability results are not constructive. It seems, therefore, necessary to consider the control problem for infinite dimensional configurations case-by-case. In this paper, I consider the control of a 3D quantum particle in a finite potential well as a first class of models considered in any physics literature on quantum systems. The controllability of such quantum systems with partly discrete and partly continuous spectrum has been partially studied in [3]. The result provided in [3], however, is far from being practical for the general case of finite potential wells of arbitrary shape. Moreover, as it is said previously the provided analysis is not constructive and does not provide a control strategy.

The simplicity of the feedback law found by the Lyapunov techniques in [2] suggests the use of the same approach for such infinite dimensional configurations. Here, we announce the main result of the paper:

Theorem 25.1.1. *Consider the Schrödinger equation*

$$\begin{aligned} &\imath \frac{\partial}{\partial t}\Psi(t,x) = -\triangle\Psi(t,x) + V(x)\Psi(t,x) + u(t)\mu(x)\Psi(t,x), \\ &\quad \Psi|_{t=0} = \Psi_0(x), \quad t \in \mathbb{R}^+, x \in \mathbb{R}^3, \quad \|\Psi_0\|_{L^2(\mathbb{R}^3)} = 1. \end{aligned} \tag{25.1}$$

We suppose the potential $V(x)$ and the dipole moment $\mu(x)$ to be bounded real-valued functions with compact supports.

We consider moreover the following assumptions:

A1 $\Psi_0 = \sum_{i=0}^{N} \alpha_i \phi_i$ *where* $\{\phi_i\}_{i=0}^{N}$ *are different normalized eigenstates in the discrete spectrum of* $H = -\triangle + V(x)$.

A2 *the coefficient* α_0 *corresponding to the population of the eigenstate* ϕ_0 *in the initial condition* Ψ_0 *is non-zero:* $\alpha_0 \neq 0$.

A3 *the Hamiltonian* $H = -\triangle + V(x)$ *admits non-degenerate transitions:* $\lambda_{i_1} - \lambda_{j_1} \neq \lambda_{i_2} - \lambda_{j_2}$ *for* $(i_1, j_1) \neq (i_2, j_2)$ *and where* $\{\lambda_i\}_{i=0}^{N}$ *are different eigenvalues of the Hamiltonian* H*;*

A4 *the interaction Hamiltonian* $\mu(x)$ *ensures simple transitions between all eigenstates of* H*:* $\langle \mu\phi_i \mid \phi_j \rangle \neq 0 \qquad \forall i \neq j \in \{0, 1, ..., N\}$.

Then for any $\epsilon > 0$*, using the feedback law* ($c > 0$)

$$u(t) = u_\epsilon(\Psi(t)) = c[(1-\epsilon)\sum_{i=0}^{N} \Im(\langle \mu\Psi \mid \phi_i \rangle \langle \phi_i \mid \Psi \rangle) + \epsilon \Im(\langle \mu\Psi \mid \phi_0 \rangle \langle \phi_0 \mid \Psi \rangle)],$$

the system admits a unique strong solution in $L^2(\mathbb{R}^3; \mathbb{C})$*. Moreover the state of the system ends up reaching a population more than* $(1-\epsilon)$ *in the eigenstate* ϕ_0 *(approximate stabilization):* $\liminf_{t\to\infty} |\langle \Psi(t,x) \mid \phi_0(x) \rangle|^2 > 1 - \epsilon$.

Remark 25.1.2. This result is perfectly comparable with the one provided for the finite dimensional configuration in [2]. However, many remarks allowing us to weaken or to remove the assumptions in the Theorem are provided in the paper. In particular, the general case of rapidly decaying potentials $V(x)$ can be addressed similarly. The assumptions **A2**,**A3** and **A4** can be alleged exactly as in the finite dimensional case. Finally, concerning the restrictive assumption **A1**, an argument based on the use of quantum adiabatic theory permits us to consider a much larger class of initial states containing an important part of the continuous spectrum.

Remark 25.1.3. Note that, even for the case of an initial state in the discrete part of the spectrum, the convergence analysis used for the finite dimensional configurations is not enough to prove the result of the Theorem. In fact, one needs to prevent the L^2-mass lost phenomena, through the continuous part of the spectrum, while stabilizing the system in the desired equilibrium state. The particular control law in the Theorem, together with some dispersive estimates of the Strichartz type, ensures this fact.

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26

Past, future, and full behaviors of passive state/signal systems

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Abstract

We describe different types of behaviors associated with a discrete time state/signal system.

Keywords

State/signal system, behavior, input map, output map, Hankel map.

In this lecture we first present an overview of the recently developed theory of passive and conservative linear time-invariant s/s (= state/signal) systems in discrete time. Such a system has a state space $\mathcal{X}$ similar to the one of a classical i/s/o (= input/state/output) system, but a s/s system differs from an i/s/o system in the sense that a s/s system does not distinguish between inputs and outputs. Instead the interaction with the surroundings takes place through a Krein signal space $\mathcal{W}$. A s/s system is *passive* if the subspace V of $\mathfrak{K}$ which generates the trajectories of the system is maximal nonnegative, and it is conservative if V is Lagrangean in $\mathfrak{K}$. A s/s system does not have just one transfer function but many transfer functions, which depending on the point of view of an outside observer can be of Schur type (from a scattering perspective), or of Carathéodory type (from an impedance perspective), or of Potapov type (from a transmission perspective).

In the time domain the standard i/o (input/output) map of an i/s/o system is replaced by a signal behavior. In [1]–[5] we defined a behavior to be a closed right-shift invariant subspace of $\ell^2(0,\infty;\mathcal{W})$. Below we shall refer to this type of behavior as a *future behavior*. If Σ is a passive s/s system (or more generally, an LFT-stabilizable s/s system), then the graph of the *Toeplitz operator* of an arbitrary i/s/o representation of Σ does not depend on the particular representation. We call this the *future behavior induced by* Σ. A future behavior is *passive* if it is a maximal nonnegative subspace of $\ell^2(0,\infty;\mathcal{W})$ induced by a fundamental decomposition of $\mathcal{W}$. The future behavior of a passive system is passive, and conversely, every passive future behavior has passive and even conservative s/s realizations.

The above definition of the future behavior is based on the Toeplitz operator of an i/s/o representation of a s/s system. The Toeplitz operator is the compression to the present and future time of the bilaterally shift-invariant *i/o map* of the i/s/o system. In many instances in system theory it is also important to study this bilaterally shift-invariant i/o map directly as well as its compression to past time, which we shall refer to as the *anti-Toeplitz* operator. We call the graphs of these two operators the *full behavior* and the *past behavior*, respectively.

In this talk we discuss the connections between past, full, and future behaviors of the original s/s system and its dual. We also introduce the notions of the *input map*, the *output map*, and the *Hankel operator* of a passive s/s system. The domain of definition of the input map and the Hankel operator is the past behavior of the system, wereas the output map is defined on the full state space. The input map is single-valued, the output map and the Hankel operator are multi-valued, and the Hankel operator is the product of the input map and the output map.

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27

Strong stabilization of almost passive linear systems

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Abstract

In this talk the stabilization of almost impedance passive systems by positive static output feedback is studied.

Keywords

System nodes, impedance passive systems, scattering passive systems, exponential and strong stability.

27.1 Introduction

The plant to be stabilized is a system node Σ. A *system node* Σ with input space U, state space X and output space Y (all Hilbert spaces) is determined by its generating triple (A, B, C) and its transfer function $\mathbf{G}$, where the operator $A : \mathcal{D}(A) \to X$ is the generator of a strongly continuous semigroup of operators $\mathbb{T}$ on X and the possibly unbounded operators B and C are such that $C : \mathcal{D}(A) \to Y$ and $B^* : \mathcal{D}(A^*) \to U$. There are no well-posedness assumptions for a system node; in particular B, C are not assumed to be admissible.

The system node Σ is called *impedance passive* if $Y = U$ and for all input functions $u \in C^2([0,\infty), U)$, and for initial states $z_0 \in X$ that satisfy $Az_0 + Bu(0) \in X$ and for all $\tau > 0$, the following holds

$$\|z(\tau)\|^2 - \|z_0\|^2 \le 2\int_0^\tau \mathrm{Re}\langle u(t), y(t)\rangle \mathrm{d}\, t.$$

Σ is called almost *impedance passive* if there exists an $E = E^* \in \mathcal{L}(U)$ such that the system node Σ_E with the same generating operators A, B, C but the transfer function $\mathbf{G} + E$ is impedance passive.

A trivial case is when $\mathbf{G}$ is already impedance passive and a special case is when Σ has colocated sensors and actuators on the boundary. Such systems include many wave and beam equations with sensors and actuators on the boundary. Characterizations of impedance passive systems have been given in Staffans [5] and from these we deduce some simpler easily verifiable conditions for systems to be impedance passive. For example, if A generates a contraction semigroup, 0 is in the resolvent set of A, and $B^*A^{*-1} = -CA^{-1}$, then Σ impedance passive if and only if $\mathbf{G}(0) + \mathbf{G}(0)^* \geq 0$. Moroever, Σ_E is almost impedance passive for all bounded self-adjoint operators $E \in \mathcal{L}(U)$ such that $E \geq -\frac{1}{2}(\mathbf{G}(0) + \mathbf{G}(0)^*)$.

It has been shown for many particular cases that the feedback $u = -\kappa y + v$, with $\kappa > 0$, stabilizes Σ, strongly or even exponentially (see [2], [4], [3]). Here, y is the output of Σ and v is the new input.

Our main result is that if $i\omega$ is in the resolvent set of A, $C(\omega I - A)^{-1} = B^*(\omega I + A^*)^{-1}$, and Σ is approximately observable and approximately controllable, then for sufficiently small k the closed-loop system is weakly stable. If, moreover, $\sigma(A) \cap i\mathbb{R}$ is countable, then the closed-loop semigroup and its dual are both strongly stable. This complements earlier results on exponential stabilization in [1], [7]. We use our results to examine the effect of feedthrough and static output feedback on large classes of damped second order PDE systems.

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Controllability, observability, stabilizability, well-posedness

28

Lur'e feedback systems with both unbounded control and observation: well–posedness and stability using nonlinear semigroups

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Abstract

We give a complement of information to Grabowski and Callier [2]. A SISO Lur'e feedback control system consisting of a linear, infinite-dimensional system of boundary control in factor form and a nonlinear static incremental sector type controller is considered. Well-posedness and a criterion of absolute strong asymptotic stability is obtained using a novel nonlinear semigroup approach.

Keywords

infinite–dimensional Lur'e feedback systems, nonlinear semigroups, stability

Consider the Lur'e feedback control system in Figure 28.1, which consists of a linear

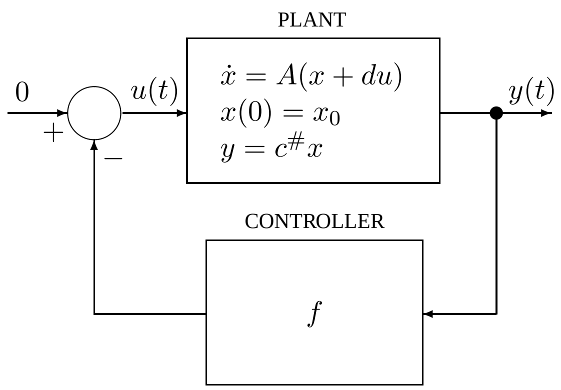

Figure 28.1: Lur'e feedback system

part described by

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & A[x(t) + du(t)] \\ y(t) & = & c^{\#}x(t) \end{array} \right\} , \tag{28.1}$$

and a scalar static controller nonlinearity $f : \mathbb{R} \to \mathbb{R}$. It is assumed that:

- $A : (\mathcal{D}(A) \subset \mathrm{H}) \longrightarrow \mathrm{H}$ generates a linear exponentially stable (**EXS**), C_0–semigroup $\{S(t)\}_{t\geq 0}$ on a Hilbert space H with a scalar product $\langle \cdot, \cdot \rangle_{\mathrm{H}}$,
- y is a scalar output defined by an A–bounded linear observation functional $c^{\#}$ (bounded on $\mathcal{D}_A$, i.e the space $\mathcal{D}(A)$ equipped with the graph norm of A, here equivalent to $\|x\|_A := \|Ax\|_{\mathrm{H}}$). The restriction of $c^{\#}$ to $\mathcal{D}(A)$ is representable as $c^{\#}x = \langle h, Ax \rangle_{\mathrm{H}}$ for every $x \in \mathcal{D}(A)$ and some $h \in \mathrm{H}$, or shortly $c^{\#}\big|_{\mathcal{D}(A)} = h^*A$.
- $d \in \mathcal{D}(c^{\#}) \subset \mathrm{H}$ is a factor control vector, $u \in \mathrm{L}^2(0, \infty)$ is a scalar control function.

The closed–loop system is described by the abstract nonlinear differential equation

$$\dot{x}(t) = A\left\{x(t) - df\left[c^{\#}x(t)\right]\right\} \tag{28.2}$$

We give conditions under which the closed -loop system operator of the right-hand side of (28.2), namely

$$\mathcal{A}x := A\left[x - df(c^{\#}x)\right], \quad \mathcal{D}(\mathcal{A}) = \left\{x \in \mathcal{D}(c^{\#}) : x - df(c^{\#}x) \in \mathcal{D}(A)\right\}, \tag{28.3}$$

is dissipative and hence the generator of a well-defined nonlinear semigroup giving that that the closed-loop system is well-posed: essentially an incremental sector type condition for the nonlinearity of the form

$$-\infty < k_1 < \frac{f(y_1) - f(y_2)}{y_1 - y_2} < k_2 < \infty \quad \forall\, y_1, y_2 \in \mathbb{R}, \qquad f(0) = 0$$

and the satisfaction of an operator Lur'e type inequality based on k_1, k_2 and the linear subsystem parameters. The solution of the latter is discussed by a circle criterion type result, essentially the condition

$$1 + (k_1 + k_2)\,\mathrm{Re}\big[\hat{g}(j\omega)\big] + k_1k_2\big|\hat{g}(j\omega)\big|^2 \geq \eta > 0 \qquad \forall \omega \in \mathbb{R}$$

where $\hat{g}$ in $\mathrm{H}^\infty(\mathbb{C}^+)$ is the transfer function of the linear subsystem. If the latter criterion is satisfied in addition to the incremental sector condition, then one gets that the state $x = 0$ of (28.2) is strongly globally asymptotically stable.

A "non–coercive" version of the stability criterion involves the Lasalle invariance principle as in Dafermos and Slemrod [1]–see [3] for more detail.

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29

A sharp geometric condition for the exponential stabilizability of a square plate by moment feedbacks only

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Abstract

We consider a boundary stabilization problem for the plate equation in a square. The feedback law gives the bending moment on a part of the boundary as function of the velocity field of the plate. The main result of the paper asserts that the obtained closed loop system is exponentially stable if and only if the controlled part of the boundary contains a vertical and a horizontal part of non zero length (the geometric optics condition introduced by Bardos, Lebeau and Rauch for the wave equation is thus not necessary in this case). The proof of the main result uses the methodology introduced in Ammari and Tucsnak [1] and a result in [2].

Keywords

Boundary stabilization, Dirichlet type boundary feedback, plate equation

29.1 Introduction and main results

In this work we study the boundary stabilization of a square Euler-Bernoulli plate by means of a feedback acting on the bending moment on a part of the boundary. Let us first describe the open loop control problem. Let $\Omega \subset \mathbb{R}^2$ be an open bounded set representing the domain occupied by the plate. We denote by $\partial\Omega$ the boundary of Ω and we assume that $\partial\Omega = \overline{\Gamma}_0 \cup \overline{\Gamma}_1$, where $\Gamma_0,\ \Gamma_1$ are open subsets of $\partial\Omega$ with $\Gamma_0 \cap \Gamma_1 = \emptyset$. The system modelling the vibrations of the plate with boundary control acting on the moment can be written as

$$\ddot{w} + \Delta^2 w = 0, \qquad x \in \Omega,\ t > 0, \tag{29.1}$$

$$w(x,t) = 0, \qquad x \in \partial\Omega,\ t > 0, \tag{29.2}$$

$$\Delta w(x,t) = 0, \qquad x \in \Gamma_0,\ t > 0 \tag{29.3}$$

$$\Delta w(x,t) = u(x,t), \qquad x \in \Gamma_1,\ t > 0 \tag{29.4}$$

$$w(x,0) = w_0(x), \quad \dot{w}(x,0) = w_1(x), \qquad x \in \Omega, \tag{29.5}$$

where we have denoted by a dot differentiation with respect to the time t and ν stands for the unit normal vector of $\partial\Omega$ pointing towards the exterior of Ω.

The main result concerns a system obtained by giving the input u in (29.4) as function of $\dot{w}$. More precisely, we consider the equations (29.1)-(29.5) by giving the control u in the feedback form

$$u(x,t) = -\frac{\partial}{\partial\nu}(G\dot{w}), \quad x \in \Gamma_1, \quad t > 0. \tag{29.6}$$

The operator G in (29.6) is defined as A_0^{-1}, where $A_0 : H_0^1(\Omega) \to H^{-1}(\Omega)$ is defined by $A_0\varphi = -\Delta\varphi$ for all $\varphi \in H_0^1(\Omega)$. Assume that Ω is a square. Moreover, suppose that $w_0 \in H_0^1(\Omega)$ and that $w_1 \in H_0^{-1}(\Omega)$. Then the initial and boundary value problem (29.1)-(29.5) determine a well posed linear dynamical system with state space $H_0^1(\Omega) \times H^{-1}(\Omega)$. We show that if Ω is a square we only need a much smaller control region. More precisely, the main results of this is the following.

Theorem 29.1.1. *Assume that Ω is a square. Then the following assertions are equivalent:*

1. *The linear dynamical system determined by* (29.1)-(29.5) *is exponentially stable in* $H_0^1(\Omega) \times H^{-1}(\Omega)$.
2. Γ_1 *contains both a horizontal and a vertical segment of non zero length.*

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30

Fast and strongly localized observation for the Schrödinger equation

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Keywords

Boundary exact observability, boundary exact controllability, Schrödinger equation, plate equation, non harmonic Fourier series, sieve methods.

30.1 Statement of the problem

In the first part of this work we study the exact observability of systems governed by the Schrödinger equation in a rectangle with homogeneous Dirichlet (respectively Neumann) boundary conditions and with Neumann (respectively Dirichlet) boundary observation. Generalizing results from Ramdani, Takahashi, Tenenbaum and Tucsnak [5], we prove that these systems are exactly observable in *in arbitrarily small time*. Moreover, we show that the above results hold even if the observation regions have *arbitrarily small measures*. More precisely, we prove that in the case of homogenous Neumann boundary conditions with Dirichlet boundary observation, the exact observability property holds for every observation region which has non empty interior. In the case of homogenous Dirichlet boundary conditions with Neumann boundary observation, we show that the exact observability property holds if and only if the observation region has an open intersection with an edge of each direction. We also show that similar results hold for the Euler-Bernoulli plate equation. Finally, we give explicit estimates for the blow-up rate of the observability constants as the time and (or) the size of the observation region tend to zero. From a qualitative point of view, the above described results essentially amount to the statement that, for any given $u, v \in]0, \infty[$ and any non empty open set $\mathcal{U} \subset \mathbb{R}^2$, there exists $\delta = \delta(\mathcal{U}) = \delta(\mathcal{U}; u, v) > 0$ such that,

$$\int_{\mathcal{U}} \Big| \sum_{m,n\in\mathbb{Z}^2} a_{mn}\, e^{2\pi i(nx+(um^2+vn^2)t} \Big|^2 \mathrm{d}x\,\mathrm{d}t \;\geq\; \delta(\mathcal{U}) \sum_{m,n\in\mathbb{Z}^2} |a_{mn}|^2$$

for all sequences $(a_{mn}) \in \ell^2(\mathbb{Z} \times \mathbb{Z}, \mathbb{C})$. This, in turn, is shown by deriving an effective version of an inequality of Beurling and Kahane and by obtaining quantitative estimates for the number of lattice points in the neighbourhood of an ellipse. The latter are obtained via techniques from analytic number theory (sieve methods).

30.2 Improvement of some estimates

The second part of this work is devoted to some improvements of recent estimates (see Miller [2], [3],[1] [4]) on the norm of the operator associating to any initial state the minimal norm control driving the system to zero. More precisely, we show that the following result holds.

Theorem 30.2.1. *Let $a > 0$, $p \in C^2[0, a]$ and $q \in C[0, a]$. Assume that $p(x) > 0$ for all $x \in [0, a]$ and denote $l = \int_0^a \sqrt{p(x)}\, \mathrm{d}x$. Let $\tau > 0$ and $\alpha > 1$. Then, for every every $z_0 \in H^{-1}(\Omega)$ there exists $u \in L^2(0, \tau)$, with*

$$\|u\|_{L^2(0,\tau)} \ll_{\tau,\alpha} e^{\frac{\alpha l^2}{\tau}} \|z_0\|_{H^{-1}(\Omega)} \qquad (z_0 \in H^{-1}(\Omega))$$

such that the solution z of

$$\begin{cases} i\dfrac{\partial z}{\partial t}(x,t) = \dfrac{\partial}{\partial x}\left(p(x)\dfrac{\partial z}{\partial x}(x,t)\right) + q(x)\, z(x,t), & x \in (0,a),\ t \geq 0 \\ z(0,t) = u(t), & t \geq 0 \\ z(a,t) = 0, & t \geq 0 \\ z(x,0) = z_0(x), & x \in (0,a), \end{cases}$$

satisfies $z(x, \tau) = 0$ for all $x \in (0, a)$.

The above result improves Theorem 4.1 in [3], where a similar assertion has been proven for $\alpha > 4\left(\dfrac{36}{37}\right)^2$.

Finally, the above results are used, following [4], to deal with the case of several space dimensions.

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31

Exact controllability of Schrödinger type systems

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Abstract

We show that if a well-posed system is described by the second order (uncontrolled) equation $\ddot{w} = -A_0 w$ and either $y = C_1 w$ or $y = C_0 \dot{w}$ (y being the output signal) and if this system is exactly observable, then this property is inherited by the system described by the first order equation $\dot{z} = iA_0 z$, with either $y = C_1 z$ or $y = C_0 z$. Such results can be used to prove the exact observability of systems governed by the Schrödinger equation, using results available for systems governed by the wave equation.

Keywords

Second order system, Schrödinger equation, exact observability.

31.1 Statement of the problem

Let H be a Hilbert space, $A_0 : \mathcal{D}(A_0) \to H$ is strictly positive and for all $\alpha > 0$, $H_\alpha = \mathcal{D}(A_0^\alpha)$ with the usual norm. Define $X = H_{\frac{1}{2}} \times H$, which is a Hilbert space with the product norm and $\mathcal{D}(A) = H_1 \times H_{\frac{1}{2}}$. Define $A : \mathcal{D}(A) \to X$ by

$$A = \begin{bmatrix} 0 & I \\ -A_0 & 0 \end{bmatrix}, \quad \text{i.e.,} \quad A \begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} g \\ -A_0 f \end{bmatrix}. \tag{31.1}$$

It is easy to see that A is skew-adjoint. X_1 stands for $\mathcal{D}(A)$ endowed with the graph norm. Our first result concerns the admissibility for observations acting on the first component of the state of the system: this admissibility is inherited by a certain Schrödinger type system.

Proposition 31.1.1. *Let Y be a Hilbert space, let $C_1 \in \mathcal{L}(H_1, Y)$ and define $C \in \mathcal{L}(X_1, Y)$ by*

$$C \;=\; [C_1\ 0]\ . \tag{31.2}$$

Assume that C is an admissible observation for the unitary group $\mathbb{T}$ generated by A. Let $\mathbb{S}$ be the unitary group generated by iA_0 on $H_{\frac{1}{2}}$. Then C_1 is an admissible observation operator for $\mathbb{S}$.

When we say that (A, C) is exactly observable in time τ, then it is understood that C is an admissible observation operator for the semigroup generated by A.

Theorem 31.1.2. *With the assumptions in Proposition* 31.1.1*, assume that the pair (A, C) is exactly observable in some positive time. Then the pair (iA_0, C_1), with the state space $H_{\frac{1}{2}}$, is exactly observable in any positive time.*

Now we consider systems where the observation acts on the second component of the state, deriving similar results. We start again with admissibility.

Proposition 31.1.3. *Let Y be a Hilbert space, let $C_0 \in \mathcal{L}(H_{\frac{1}{2}}, Y)$ and define $C \in \mathcal{L}(X_1, Y)$ by*

$$C \;=\; [0\ C_0]\ . \tag{31.3}$$

Assume that C is an admissible observation for the unitary group $\mathbb{T}$ generated by A. Let $\mathbb{S}$ be the unitary group generated by iA_0 on H. Then C_0 is an admissible observation operator for $\mathbb{S}$.

Now comes the corresponding controllability result:

Theorem 31.1.4. *With the assumptions in Proposition* 31.1.3*, assume that the pair (A, C) is exactly observable in some positive time. Then the pair (iA_0, C_0), with state space H, is exactly observable in any positive time.*

We mention that under a certain assumption on the spectrum of A_0, the converses of the above theorems are also true. For the proofs and for other details (examples) we refer to Chapter 5 of our book [1].

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32

Controllability of the nonlinear Korteweg-de Vries equation for critical spatial lengths

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Abstract

It is known that the linear Korteweg-de Vries equation with homogeneous Dirichlet boundary conditions and Neumann boundary control is not controllable for some critical spatial domains. In this paper, we prove for these critical cases, that the nonlinear equation is locally controllable around the origin provided that the time of control is large enough. It is done by performing a power series expansion of the solution and studying the cascade system resulting of this expansion.

Keywords

controllability, Korteweg-de Vries equation, critical domains, power series expansion

32.1 Introduction

Let $L > 0$ be fixed. Let us consider the following Neumann boundary control system for the Korteweg-de Vries (KdV) equation with the Dirichlet boundary condition

$$\begin{cases} y_t + y_x + y_{xxx} + yy_x = 0, \\ y(t,0) = y(t,L) = 0, \\ y_x(t,L) = u(t), \end{cases} \tag{32.1}$$

where the state is $y(t,\cdot) : [0,L] \to \mathbb{R}$ and the control is $u(t) \in \mathbb{R}$. In this paper, we are concerned with the controllability of (32.1). More precisely, for a time $T > 0$, we want to prove the following property.

Property 32.1.1. *(Local exact controllability)* There exists $r > 0$ such that, for every $(y_0, y_T) \in L^2(0, L)^2$ with $\|y_0\|_{L^2(0,L)} < r$ and $\|y_T\|_{L^2(0,L)} < r$, there exist $u \in L^2(0, T)$ and

$$y \in C([0, T], L^2(0, L)) \cap L^2(0, T, H^1(0, L))$$

satisfying *(32.1)*, $y(0, \cdot) = y_0$ and $y(T, \cdot) = y_T$.

In order to deal with the nonlinear term in (32.1), one can perform a power series expansion of (y, u).

In [3] Rosier has studied the control system (32.1) by using a first order expansion, i.e. he considered the linear control system. He proved that the linear KdV system is exactly controllable and the nonlinear one is exactly locally controllable provided that

$$L \notin \mathcal{N} := \left\{ 2\pi\sqrt{\frac{k^2 + kl + l^2}{3}};\, k, l \in \mathbb{N}^* \right\}. \tag{32.2}$$

If $L \in \mathcal{N}$, Rosier proved that there exists a finite-dimensional subspace of $L^2(0, L)$, denoted by M, which is unreachable for the linear system.

In [2] Coron and Crépeau studied the first case i.e M is one-dimensional. First, they prove that one can reach all the missed directions lying in M with a third order power series expansion and then they demonstrate that Property 32.1.1 holds true ([2, Theorem 2]).

In [1] Cerpa uses the same approach to treat the second critical case: M is two-dimensional and a second order expansion allows to enter into the subspace M. If the time of control is large enough, one can reach all the missed direction. By using this fact and a fixed point argument one obtains Property 32.1.1 provided that T is large enough ([1, Theorem 1.4]).

32.2 Main result

By using results of [3, 1, 2] we prove that Property 32.1.1 holds in other critical cases, i.e. when the dimension of the subspace M is higher than 2. We use an expansion to the second order if $L \neq 2\pi k$ for any $k \in \mathbb{N}^*$ and an expansion to the third order if $L = 2\pi k$ for some $k \in \mathbb{N}^*$. With particular control, constructed from controls of proposition 3.2 [1] and proposition 10 [2], we reach a basis of directions in M. We get all the other direction after a time T_L long enough, using the fact that in M, with no control, the solution only turns with a known celerity. Then using two fixed point theorems similar to those used in [2, 1], we get the main result of this work.

Theorem 32.2.1. Let $L \in \mathcal{N}$. Then, there exists $T_L > 0$ such that Property 32.1.1 holds provided that $T > T_L$.

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Properties of linear systems

33

Well-posedness and regularity of hyperbolic systems

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Abstract

We show that a hyperbolic partial differential equation with control and observation at the boundary of a one-dimensional spatial domain is well-posed if and only if the homogeneous equation, i.e., the input set to zero, is well-defined.

Keywords

Hyperbolic partial differential equation, well-posedness, regularity.

33.1 Introduction

Consider the well-known wave equation

$$\frac{\partial^2 w}{\partial t^2}(x,t) = c\frac{\partial^2 w}{\partial x^2}(x,t), \tag{33.1}$$

where $c = \frac{T}{\rho}$, with T Young's modulus and ρ the mass density. We can write this as

$$\begin{aligned}\frac{\partial}{\partial t}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}(x,t) &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{\partial}{\partial x}\begin{pmatrix} \frac{1}{\rho}z_1 \\ Tz_2 \end{pmatrix}(x,t) \\ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{\partial}{\partial x}\left[\begin{pmatrix} \frac{1}{\rho} & 0 \\ 0 & T \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}\right](x,t),\end{aligned}$$

where $z_1(x,t) = \rho\frac{\partial w}{\partial t}(x,t)$, and $z_2(x,t) = \frac{\partial w}{\partial x}(x,t)$.

Like in the above example, many hyperbolic p.d.e.'s can be written in the above form. Hence we assume that our p.d.e. is of the form

$$\frac{\partial z}{\partial t} = P_1 \frac{\partial}{\partial x} (\mathcal{L}z) + P_0 z, \qquad x \in [a,b]. \tag{33.2}$$

where z is a vector valued function, and $\mathcal{L}$ is a multiplication operator which satisfies $0 < mI \leq \mathcal{L}(x) \leq MI$, for some constants m and M. With this $\mathcal{L}$ we introduce the Hilbert space Z as being the function space $L^2((a,b);\mathbb{R}^n)$ with inner product

$$\langle f, g \rangle = \int_a^b f(x)^* \mathcal{L}(x) g(x) dx.$$

Theorem 33.1.1. *Consider the partial differential equation*

$$\frac{\partial z}{\partial t}(x,t) = P_1 \frac{\partial}{\partial x} (\mathcal{L}z)(x,t) + P_0(x) z(x,t), \qquad x \in [a,b], \quad z(x,0) = z_0(x) \tag{33.3}$$

$$0 = M_{11} (\mathcal{L}z)(b,t) + M_{12} (\mathcal{L}z)(a,t) \tag{33.4}$$

$$u(t) = M_{21} (\mathcal{L}z)(b,t) + M_{22} (\mathcal{L}z)(a,t) \tag{33.5}$$

$$y(t) = C_1 (\mathcal{L}z)(b,t) + C_2 (\mathcal{L}z)(a,t) \tag{33.6}$$

where $z(x,t) \in \mathbb{R}^n$, $P_1^T = P_1$, $\operatorname{rank} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \operatorname{rank} \begin{bmatrix} M_{11} & M_{12} \end{bmatrix} + \operatorname{rank} \begin{bmatrix} M_{21} & M_{22} \end{bmatrix} = n$, $\operatorname{rank} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \\ C_1 & C_2 \end{bmatrix} = n + \operatorname{rank} \begin{bmatrix} C_1 & C_2 \end{bmatrix}$*, and* $\mathcal{L}$ *satisfies the condition stated above. If the homogeneous p.d.e., i.e.,* $u \equiv 0$*, generates a* C_0*-semigroup on* Z*, then the system (33.3)–(33.6) is well-posed, and the corresponding transfer function is regular.*

Well-posedness means that there exists an $m_f > 0$ and $t_f > 0$ such that for all smooth initial conditions and inputs the following holds

$$\|z(t_f)\|^2 + \int_0^{t_f} \|y(t)\|^2 dt \leq m_f \left[\|z_0\|^2 + \int_0^{t_f} \|u(t)\|^2 dt \right]. \tag{33.7}$$

The proof is based on the work in [1] combining it with the feedback result of Weiss [2]. A preliminary version of this theorem has been published in [3]. Note that in [1] necessary and sufficient conditions were given such that the homogeneous p.d.e. generates a contraction semigroup. Hence from our theorem we conclude that all these systems are well-posed and regular.

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Casimir functions and interconnection of boundary port-Hamiltonian systems

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Abstract

It is known that Casimir functions can be used for energy shaping of finite dimensional Hamiltonian systems. As a first step towards the generalization to boundary port Hamiltonian systems, we define a Poisson bracket and characterize the Casimir functions for Dirac structures arizing in a class of boundary port Hamiltonian systems [6]. We also analyze the Casimir functions of mixed systems composed of a boundary port Hamiltonian system coupled with two finite-dimensional port Hamiltonian systems [3], [4] .

Keywords

Boundary control systems, Hamiltonian systems, Poisson bracket, Casimir functions.

In this paper we consider skew-symmetric differential operator of the form:

$$\mathcal{J} = P_1 \frac{\partial}{\partial z} \text{ where } P_1 \in \mathbb{R}^{n,n} \qquad \text{with } P_1 = P_1^T \tag{34.1}$$

Following [6], we can define a set of boundary port variables (e_∂, f_∂) such that the subspace $\mathcal{D} \ni (f, f_\partial, e, e_\partial) \subset L_2(a,b;\mathbb{R}^n) \times \mathbb{R}^n \times H^1(a,b;\mathbb{R}^n) \times \mathbb{R}^n$ defined as

$$\mathcal{D} = \left\{ \begin{pmatrix} f \\ f_\partial \\ e \\ e_\partial, \end{pmatrix} \in L_2(a,b;\mathbb{R}^n) \times \mathbb{R}^n \times H^1(a,b;\mathbb{R}^n) \times \mathbb{R}^n \mid f = \mathcal{J}e, \right.$$
$$\left. \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{bmatrix} P_1 & -P_1 \\ I & I \end{bmatrix} \begin{pmatrix} e(b) \\ e(a) \end{pmatrix} \right\} \tag{34.2}$$

is a Dirac structure with respect to the canonical symmetric pairing generated by the L_2 inner product.

Let $\mathcal{E}_{\rm adm} = \left\{ e \in H^1(a,b;\mathbb{R}^n) \mid \exists f \in L_2(a,b;\mathbb{R}^n) \text{ such that } (f,e) \in \mathcal{D} \right\}$ be the space of admissible efforts. Following [1], [2], and [5] we define on this space a skew-symmetric bracket: $[\left(\begin{smallmatrix} e_1 \\ e_{\partial 1}\end{smallmatrix}\right), \left(\begin{smallmatrix} e_2 \\ e_{\partial 2}\end{smallmatrix}\right)] = \int_a^b e_1^T \mathcal{J} e_2 - e_{\partial 1}^T P_1 \left(e_2(b) - e_2(a)\right)$. This bracket is used to define a Poisson bracket on some suitable functional space $K_{\rm adm}$, satisfying: $\{k_1, k_2\}(a) := [\delta k_1(a), \delta k_2(a)]$, $k_1, k_2 \in K_{\rm adm}$, where δ denotes the variational derivative.

In the second part, we investigate the Casimir functions associated with the previously defined Poisson bracket. These Casimir functions are functions $C \in K_{\rm adm}$ such that $\{k, C\} = [\delta k, \delta C] = 0, \ \forall k \in K_{\rm adm}$. We show that the Casimir functions are the same functions of the state variables as the Casimir functions associated with the Poisson bracket on $\mathbb{C}^n$ with structure matrix $J = iP_1$ on the finite dimensional space $\mathbb{C}^n$.

In the third part, we consider the bracket arising from the interconnection of a port-boundary Hamiltonian system with two finite-dimensional port Hamiltonian systems at its boundaries [3], [4]. In a first instance we shall consider the case of $P_1 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0\end{smallmatrix}\right)$ (arising for the transmission line or the vibrating string models). We derive the Casimir functions of the total system with respect to the Casimir functions of the two finite dimensional systems. In particular, we show that if the two finite-dimensional systems have no Casimirs, then there exits a Casimir function for the total system corresponding to topological invariants such as Kirchhoff's mesh law.

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Compactness of the difference between two thermoelastic semigroups

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Abstract

Our goal is to prove the compactness of the difference between the thermoelasticity semigroup and its decoupled semigroup. To show this, we prove the norm continuity of this difference, the compactness of the difference of the resolvents of these semigroups and use a result of Li-Gu-Huang. An example of thermoelastic systems with Neumann Laplacian on a Jelly Roll domain is given.

Keywords

thermoelasticity, semigroup, compactness, norm continuity and fractional powers

35.1 Introduction

Consider the classical abstract thermoelasticity system

$$(1) \quad \begin{cases} u_{tt} + Au + B\theta &= 0, \quad t \geq 0, \\ \theta_t + C\theta - B^* u_t &= 0, \quad t \geq 0, \end{cases}$$

where $A : D(A) \subset H_1 \longrightarrow H_1$ and $C : D(C) \subset H_2 \longrightarrow H_2$ are self adjoint positive operators with bounded inverses (not necessarily compact), while $B : D(B) \subset H_2 \longrightarrow H_1$ is a closed operator with adjoint B^*, such that $D(C^{\frac{1}{2}}) \subset D(B)$ and $D(A^{\frac{1}{2}}) \subset D(B^*)$. The asymptotic behavior of this system has been studied by several authors see, [1, 2, 3, 5, 7], by the decoupling technic. Namely, they consider the simpler system

$$(2) \quad \begin{cases} u_{tt} + Au + BC^{-1}B^* u_t &= 0, \quad t \geq 0, \\ \theta_t + C\theta - B^* u_t &= 0, \quad t \geq 0, \end{cases}$$

and they proved that the difference between the semigroups $(T(t))$ and $(T_d(t))$ generated by these two systems is compact (then $\sigma_{ess}(T(t)) = \sigma_{ess}(T_d(t))$), under the compactness of $BC^{-\gamma}$ for some $0 < \gamma < 1$.

In this paper, we obtain the same result under weaker conditions and following a different approach. For this we show the following lemma.

Lemma 35.1.1. *(i)* $t \longmapsto T(t) - T_d(t)$ *is norm continuous in* $(0, \infty)$.
(ii) Assume that $A^{-1}BC^{-1}$ *is compact. Then* $R(\lambda, L) - R(\lambda, L_0)$ *is compact for every* $\lambda \in \rho(L) \cap \rho(L_0)$, *where* L_0 *and* L *are the generators of* $T(\cdot)$ *and* $T_d(\cdot)$ *respectively.*

Hence, [4, Theorem 2.3] leads to our aim.

Theorem 35.1.2. *Assume that* $A^{-1}BC^{-1}$ *is compact. Then* $T_d(t) - T(t)$ *is compact for all* $t \geq 0$.

At the end, we illustrate our generalization by the following thermoelastic system on a special bounded domain, proposed in [6], $\Omega = \{(x, y) \in \mathbb{R}^2 : \frac{1}{2} < r < 1\} \setminus \Gamma$, where Γ is the curve, in $\mathbb{R}^2$, given in polar coordinates by
$r(\phi) = \dfrac{\frac{3\pi}{2} + \text{Arctang}(\phi)}{2\pi}, \quad -\infty < \phi < \infty$. For this system, we show that

Proposition 35.1.3. $A^{-1}BC^{-1}$ *is a compact operator but the operator* $BC^{-\gamma}$ *is not compact for every* $0 < \gamma < 1$.

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36

On nonexistence of maximal asymptotics for certain linear equations in Banach space

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Abstract

This work continues the analysis of the certain asymptotic behavior of the solutions of certain linear differential equations in Banach space originated in [1] and developed in [2, 3] (see also [4] and references therein).

36.1 Statement of the problem

We consider the equation

$$\dot{x} = Ax, \quad x \in X, \tag{36.1}$$

where X is a Banach space, assuming that A is an infinitesimal operator generating the C_0-semigroup denoted by $\left\{e^{At}\right\},\ t \geq 0$. We also assume that $\left\|e^{At}\right\| > 0,\ t \geq 0$.

Definition 36.1.1. We say that the equation (36.1) (or the semigroup $\left\{e^{At}\right\},\ t \geq 0$) has a maximal asymptotics if there exists a real positive function, say $f(t),\ t \geq 0$, such that

i) for any initial vector $x \in X$ the function $\left\|e^{At}x\right\| / f(t)$ is bounded on $[0, +\infty]$,

ii) there exists at least one $x_0 \in X$ such that

$$\lim_{t \to +\infty} \frac{\left\|e^{At}x_0\right\|}{f(t)} = 1.$$

Note that in the finite-dimensional case the maximal asymptotics always exists. More exactly a function $f(t)$ from Definition 36.1.1 may be chosen as

$$f(t) = t^{p-1}e^{\mu t},$$

where $\mu = \max\limits_{\lambda \in \sigma(A)} \operatorname{Re} \lambda$ and p is the maximal size of Jordan boxes corresponding to the eigenvalues of A with real part equal μ. In the infinite-dimensional case it is relatively easy to give an example of equation (even with bounded A) for which the maximal asymptotics does not exist. In this context the main result of [1] may be interpreted in the following way:

Let the semigroup $\{e^{At}\}$, $t \geq 0$ be bounded and let $\sigma(A) \cap (i\mathbb{R})$ be at most countable set. Then the asymptotics $f(t) \equiv 1$ is maximal for this semigroup iff A^* possess a pure imaginary eigenvalue. In particular this means that if $\sigma(A) \cap (i\mathbb{R})$ is in addition nonempty but does not contain eigenvalues then the semigroup has no maximal asymptotics at all.

The main contribution of the present work are the following theorems.

Theorem 36.1.2. *Assume that*

i) $\sigma(A) \cap \{\lambda : \operatorname{Re} \lambda = \mathcal{O}_0\}$ *is at most countable,* $\mathcal{O}_0 = \lim\limits_{t \to +\infty} \dfrac{\log(\|e^{At}\|)}{t}$*;*

ii) operator A^* *does not possess eigenvalues with real part equals* $\mathcal{O}_0$.

Then the equation (36.1) *(the semigroup* $\{e^{At}\}$*,* $t \geq 0$*) does not have any maximal asymptotics.*

Theorem 36.1.3. *Let the assumptions of Theorem 36.1.2 be satisfied and let* $f(t)$*,* $t \geq 0$ *be a positive function such that:*

a) $\log f(t)$ *is concave,*

b) for any $x \in X$ *the function* $\|e^{At}x\|/f(t)$ *is bounded.*

Then

$$\lim_{t \to +\infty} \|e^{At}x\|/f(t) = 0, \quad x \in X. \tag{36.2}$$

These results find the application in estimation of asymptotics of solutions, for example, of delayed equations [5].

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Non-linear PDE's, theory and applications

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A biologically inspired synchronization of lumped parameter oscillators through a distributed parameter channel

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Abstract

A generic biologically inspired synchronization problem modeled as two Duffing oscillators exchanging synchronization solitons through a Korteweg-deVries or Klein-Gordon channel is investigated.

Keywords

Duffing oscillators, KdV equation, Klein-Gordon equation, breather solution, standing wave, synchronization

Oscillations of the spinal column in vertebrates has been widely investigated in such benchmark phenomena as the swimming of the lamprey, the crawling of the salamander, even in the electrically induced gait movement in quadriplegic subjects [4]. All of these phenomena are the manifestation of a Central Pattern Generator (CPG), a concept that involves by far deeper control theory than what its original development might have led us to believe [7]. While some Partial Differential Equation (PDE) model of spinal oscillations can be derived from neuro-physiology first principles (or even by differential algebra [8, Sec. 2.1.1] modeling from experimental surface electromyographic (sEMG) signals), the missing piece of the puzzle has been what is happening at the distal ends of the spinal column—the boundary conditions. Surprisingly enough, the "boundary conditions" are better understood for humans than vertebrate animals, as they were formulated under "dural-vertebral attachments" by the late neurosurgeon A. Breig [2]. The latter paradigm states that the cervical vertebra are mechanically attached to the spinal dura, hence creating a sensory-motor loop, itself eliciting local oscillations visible as a twitching of the neck muscles in manipulative medicine. Further manipulation then induces a hip movement; thereafter, electrophysiological waves [6] run up and down the spine, induces a chaotic-like transient, after which the neck and hip are brought in sync by a standing wave pattern along the spine. Even more, bifurcation between opposite phase oscillations under a standing wave with one mode shape node and in phase oscillations under two mode shape nodes can be observed [5]. The sEMG record shows "bursts of accrued sEMG activity" at a fundamental frequency of 150 Hz. running along the spine, colliding and *surviving the collision* in a soliton-like propagation. The persistence, the *robustness*, of this phenomenon across the population of research subjects, which even includes quadriplegic subjects, mandates some theoretical justification for it.

The spinal wave has both a lateral and a longitudinal component; however, the relevant phenomena are so far better understood in the coronal plane, so the analysis will be simplified to a 1-dimensional motion $y(x,t)$, where $x \in [0,L]$, solution of a PDE $Py(x,t) = 0$, where P is a partial differential operator subject to boundary conditions $D_0 y(0,t) = 0$, $D_L y(L,t) = 0$, where D_0, D_L are differential operators. Among the most challenging problems are the dubious accuracy of PDE models of spinal oscillation, especially in their ability to model this particular phenomenon, and the inherently noisy nature of the experimental sEMG signals, making PDE modeling from experimental data bit unreliable. Beyond these modeling uncertainties, one thing is absolutely certain—the remarkable robustness of the standing wave and its ability to bifurcate. So the problem is tackled the other way around: Identify operators P, D_0, D_L that lead to such behavior, and then proceed to confirm the latter neuropysiologically. One combination that quite nicely matches the experimental data is the breather solution $y(x,t) = 4\tan^{-1}\left(\frac{\sqrt{1-\mathcal{O}^2}}{\mathcal{O}}\sin(\mathcal{O}t)\mathrm{sech}\left(\sqrt{1-\mathcal{O}^2}x\right)\right)$ to the sine-Gordon equation $Py = 0$. Even though a bit tortuous, the Euler-Lagrange formalism of Manton still applies. This leads to a Lagrangian made up of a distributed parameter part (the communication medium) and two lumped parameters parts (the boundary oscillators). The standing wave solutions, which induces synchronization of the boundary conditions, are found as minima of the potential. The topological soliton property arises from the fact that in infinite dimension these minima cannot be destroyed [3]. (Another model involves the periodically forced Korteweg-deVries equation [1].)

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Boundary control of a channel in presence of small perturbations: a Riemann approach

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Abstract

The problem of stability of the non-linear Saint-Venant equations, written in terms of a system of two conservation laws perturbed by non-homogeneous terms, is studied. Under some assumptions on those non-homogeneous functions, previous results on the stability of two conservation laws are developed using the Riemann coordinates approach.

38.1 Model

We consider the following model of flow in open-channels (Saint-Venant equations)

$$\partial_t H + \partial_x (Q/B) = q, \quad \partial_t Q + \partial_x (\frac{Q^2}{BH} + \frac{1}{2} g B H^2) = gBH(I-J) + kq\frac{Q}{BH}, \qquad (38.1)$$

where $H(x,t)$ stands for the water level and $Q(x,t)$ the water flows in the reach while g denotes the gravitation constant. I is the bottom slope, B is the channel width and J is the slope's friction expressed with the Manning-Strickler expression. The function $q(x,t)$ stands for a lateral flow by unit length and k is a constant such that $k=0$ for supply, $k=1$ for loss.

The control actions are the positions U_0 and U_L of the two spillways located at the extremities of the pool which expressions for two submerged underflow gate at upstream and downstream are respectively:

$$Q(0,t) = U_0 B \mu_0 \sqrt{2g(z_{up} - H(0,t))}, \quad Q(L,t) = U_L B \mu_L \sqrt{2g(H(L,t) - z_{do})}, \qquad (38.2)$$

where z_{up} and z_{do} are the water levels before and after, respectively, the $i-th$ gate ($i = 0, L$). The water flow coefficient of the i-th gate is denoted μ_i.

For constant control actions $U_0(t) = \bar{U}_0$ and $U_L(t) = \bar{U}_L$, a *steady-state solution* is a constant solution $(H, Q)(x, t) = \left(\bar{H}, \bar{Q}\right)(x)$ for all $t \in [0, +\infty)$, and $x \in [0, L]$ which satisfies (38.1) and the boundary conditions (38.2).

The problem under consideration is the following: given a steady-state $\left(\bar{H}, \bar{Q}\right)$, we want to compute an *output* feedback controller $y = (H_0, H_L) \mapsto (U_0(y), U_L(y))$, with $H_0 = H(0, t)$, $H_L = H(L, t)$, such that, for any smooth small enough (in C^1 norm) initial condition $H^{\#}$ and $Q^{\#}$ satisfying some compatibility conditions, the PDE (38.1) with the boundary conditions (38.2) and the initial condition $(H, Q)(x, 0) = (H^{\#}, Q^{\#})(x)$ for all $x \in [0, L]$, has a unique smooth solution converging exponentially fast (in C^1-norm) towards $\left(\bar{H}, \bar{Q}\right)$. The boundary conditions are written as follows: $a(0, t) + k_0 b(0, t) = 0$, $b(L, t) + k_L a(L, t) = 0$, where k_0, k_L are constant design parameters that have to be tuned to guarantee the stability and a and b are the Riemann coordinates.

38.2 Main result

Theorem 38.2.1. *Let t_1, t_2, ℓ_1 and ℓ_2 four constants depending on the eigenvalues of the Jacobian matrix of the system and on the steady state (see [1]). If the bottom slope function I, the slope's friction function J and the supply function q are sufficiently small in C^1-norm, then we have $\max(t_1\ell_1, t_2\ell_2) < 1$.*
In that case, there exist k_0 and k_L such that

$$| k_0 k_L | + t_2 | k_0 | \ell_2 + t_1 \ell_1 < 1, \qquad | k_0 k_L | + t_1 | k_L | \ell_1 + t_2 \ell_2 < 1,$$

and the following boundary output feedback controller

$$U_0 = H_0 \frac{\frac{\bar{Q}_0}{B\bar{H}_0} - 2\sqrt{g}\alpha_0\left(\sqrt{H_0} - \sqrt{\bar{H}_0}\right)}{\mu_0\sqrt{2g(z_{up} - H(0,t))}}, \qquad U_L = H_L \frac{\frac{\bar{Q}_L}{B\bar{H}_L} + 2\sqrt{g}\alpha_L\left(\sqrt{H_L} - \sqrt{\bar{H}_L}\right)}{\mu_L\sqrt{2g(H(L,t) - z_{do})}}$$

where $\alpha_0 = \frac{1-k_0}{1+k_0}$, and $\alpha_L = \frac{1-k_L}{1+k_L}$ makes the closed loop system locally exponentially stable, i.e. there exist $\varepsilon > 0$, $C > 0$ and $\mu > 0$ such that, for all initial conditions $(H^{\#}, Q^{\#}) : [0, L] \to (0, +\infty)$ continuously differentiable, satisfying some compatibility conditions and the inequality $|(H^{\#}, Q^{\#}) - (\bar{H}, \bar{Q})|_{C^1(0,L)} \leq \varepsilon$,
there exists a unique C^1-solution of the Saint-Venant equations (38.1), with the boundary conditions (38.2) and the initial condition $(H, Q)(\cdot, 0) = (H^{\#}, Q^{\#})(\cdot)$, defined for all $(x, t) \in [0, L] \times [0, +\infty)$. Moreover it satisfies, $\forall t \geq 0$,

$$|(H, Q)(.t) - (\bar{H}, \bar{Q})|_{C^1(0,L)} \leq Ce^{-\mu t}|(H^{\#}, Q^{\#})|_{C^1(0,L)} .$$

This stability result is applied to the regulation problem of the water level and flow of the shallow water equation and is illustrated with numerical results using the data of a real river (namely the Sambre in Belgium, and the Gignac channel in France using the software SIC of the CEMAGREF), and experimentations on a micro-channel (more precisely the Valence experimental reach).

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Boundary control of a channel: internal model boundary control

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Keywords

Shallow water equations, infinite dimensional perturbation theory, stabilization, multivariable internal model boundary control, hyperbolic PDE.

This paper deals with the regulation problem of irrigation channels with a mono or multi-objective control. The control problem is stated as a boundary control of hyperbolic Saint-Venant Partial Differential Equations (pde) [4] (Fig. 39.1).
Regulation is done around an equilibrium state and spatial dependency of the operator parameters is taken into account in the linearized model. Previous stability results have been generalized using perturbation theory in infinite dimensional Hilbert space, including more general hyperbolic systems [2], [1], which can be written as:

$$\partial_t \xi(t) = A_d(x)\xi(t), \quad x \in \Omega, \quad t > 0 \tag{39.1}$$

$$F_b\xi(t) = B_b u(t), \text{ on } \Gamma = \partial\Omega, t > 0 \tag{39.2}$$

$$\xi(x,0) = \xi_0(x) \tag{39.3}$$

where $A_d(x) = A_e(x)\partial_x + B_e(x)$ is an hyperbolic operator, and $F_b(\xi) = F_0\xi(0,t) + F_L\xi(L,t)$. Results from [3] works, show that the abstract boundary control system (39.1)-(39.3) has a solution that exists and belongs to $D(A_d)$ if A_d is a closed, densely defined operator, and generates a C_0-semigroup. The last assumptions on the operator A_d are realized under the following hypothesis:

a) $B_e(x)$ is $A_e(x)\partial_x$-bounded with $b < 1$ on a Hilbert space ($b < 1/2$ for a Banach), and $B_e(x)$ is densely defined,

b) $-A_e(0)F_0 - A_e(L)F_L$ is invertible,

c) A_e is invertible, densely defined and A_e^{-1} is bounded.

The semigroup generated is exponentially stable if

i) $B_e(x)$ is semi-definite negative, **ii)** $0 \in \rho(A(x)) = \rho(A_e(x)\partial_x + B_e(x))$.

The Internal Model Boundary Control (IMBC) (Fig. 3) used in a direct approach allows to make a control parameters synthesis by semigroup conservation properties, like the exponential stability of the closed loop system.

Simulations using the data of a real river (namely the Sambre in Belgium, the Gignac channel in France using the software SIC of the CEMAGREF, Fig 39.2), and experimentations on the Valence experimental micro-channel show that this approach should be suitable for more realistic situations.

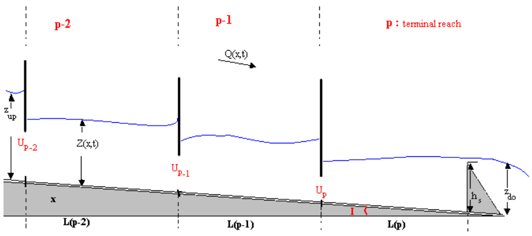

Figure 39.1: Multireach in cascade

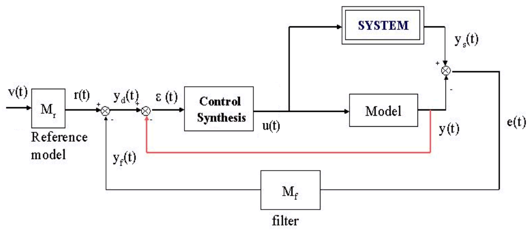

Figure 3: IMBC structure

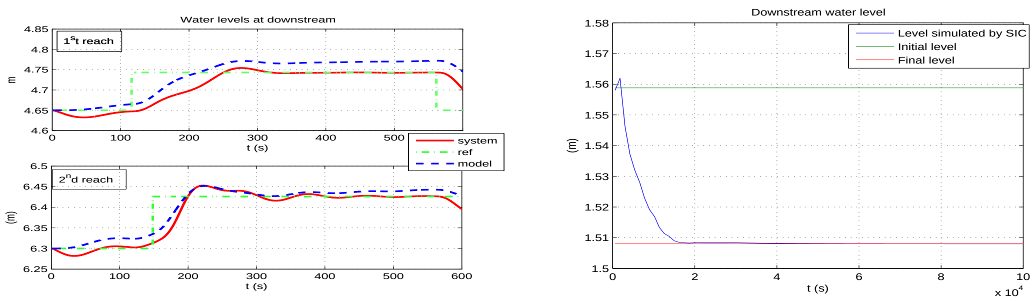

Figure 39.2: Regulation of the downstream water levels of two Sambre reaches and one Gignac reach

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Constrained adaptive control for a nonlinear distributed parameter tubular reactor

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Abstract

In this paper we present an adaptive control for a non linear distributed parameter exothermic chemical reaction in tubular reactor, this controller is presented with partial measurement. It is shown that under suitable conditions on the different parameter of the system, we can derive the temperature to a ball with pre-specific radius centred at pre-specific a profile of the temperature.

Keywords

Adaptive control, exothermic chemical reaction, λ-tracking.

40.1 Introduction

The dynamics of a nonisothermal tubular reactor with axial dispersion are described by nonlinear partial differential equations which can be transformed within the framework of the semi-linear systems as follows [1]:

$$\dot{x}_1(t) = A_1 x_1(t) + \alpha f(x_1(t), x_2(t)) + u(t) \tag{40.1}$$

$$\dot{x}_2(t) = -A_2(x_2^{in} - x_2(t)) - f(x_1(t), x_2(t)) \tag{40.2}$$

Where the operators A_1 and A_2 are:
$A_1 x = D_1 \frac{d^2x}{d^2z} - v\frac{dx}{dz} - k_0 x$ and $A_2 x = D_2 \frac{d^2x}{d^2z} - v\frac{dx}{dz}$ for $x \in H = L^2(0, L)$
With: $D(A_i) = \{x \in H : x, \frac{dx}{dz}$ are a.c. , $\frac{d^2x}{dz^2} \in H, D_i \frac{dx}{dz}(0) - vx(0) = 0, \frac{dx}{dz}(L) = 0\}$
The physical considerations lead us to assume that u is constrained so that there exist $\overline{u}$ and $\underline{u}$ with $0 < \underline{u} < \overline{u}$ such that: $\underline{u} \leq u(t) \leq \overline{u}$.
Recently, a constrained adaptive control scheme has been developed with the objective to regulate the temperature of exothermic tubular reactors in ball centred at the temperature profile

x^* and of arbitrary prescribed radius $\lambda > 0$ [1]. The implementation of this controller requires measurement of the reactor temperature over the entire spacial domain which presents a practical limitation in the case of the large reactor. To overcome this limitation we suppose in this work that we can measure the temperature reactor just in a zone $\Omega \subset [0, L]$, with L is the reactor length.
For this raison, put $C = 1\!\!1_\Omega(z)$ and $\widehat{e}(t)(.) = C(.)(x^*(.) - x_1(t)(.)) = C(.)e(t)$.
The proposed controller is called λ-tracker is given by

$$u(t) = sat_{[\underline{u},\overline{u}]}(\beta(t)\widehat{e}(t) + u^*) \qquad (40.3)$$

$$\dot{\beta}(t) = k_1 \begin{cases} (||\widehat{e}(t)(t)|| - \lambda)^l & \text{if} \quad ||\widehat{e}(t)(t)|| > \lambda \\ 0 & \text{if} \quad ||\widehat{e}(t)(t)|| \leq \lambda \end{cases} \qquad (40.4)$$

40.2 Main result

We consider the following assumptions:

- (H_1) the positif cone $H^+ \times H^+$ is positively invariant under (40.1)- (40.2) for all nonnegative control $u(.)$.
- (H_2) For $x^* > 0$ there exist $0 < \underline{x} < \overline{x}$, $\rho > 0$: such that for $0 < x_1 \leq \overline{x}$ and $0 < x_2 \leq x_2^{in}$
$$\begin{cases} \underline{u} + \rho \leq k_0 x_1 - \alpha f(x_1, x_2) - Ax^* \leq u* - \rho \leq \overline{u} - \rho \\ D_1 \frac{d^2(\overline{x} - x^*)}{d^2 z} - v \frac{d(\overline{x} - x^*)}{dz} \leq 0 \end{cases}$$
Where $Ax^* = D_1 \frac{d^2 x^*}{d^2 z} - v \frac{dx^*}{dz}$
- (H_3) $0 < \lambda < \overline{x} - x^*, \quad 0 < \underline{x} < x^* < \overline{x}$

In this work we consider local λ-control in the sense that the initial temperature $x_1(0)$ is constrained to be in the set $\Delta_1 = \{x_1 \in H / \ 0 < x_1 \leq \overline{x}\}$, we define also the set $\Delta_2 = \{x_2 \in H / 0 < x_2 \leq x_2^{in}\}$

Theorem 40.2.1. *Assume that* (H_1), (H_2) *and* (H_3) *hold, and* $(x_1^0, x_2^0) \in \Delta_1 \times \Delta_2$ *and suppose:* $\beta(0) \geq \dfrac{u^* - \underline{u}}{\overline{x} - x^*}$
the closed loop system given by equations (40.1)-(40.2) and u *given by (40.3) has the following properties:*

- $x_1(.), x_2(.), \beta(.) : I\!\!R_{\geq 0} \longrightarrow \Delta_1 \times \Delta_2 \times I\!\!R_{\geq 0}$
- $\lim_{t \longrightarrow +\infty} \beta(t)$ *exists and is finite.*
- $\limsup_{t \longrightarrow +\infty} \|\widehat{e}(t)\| \leq \lambda$

and if $mes(\overline{\Omega} = [0, L] \backslash \Omega) = \mathcal{O} < \dfrac{k\lambda^2}{\|\overline{x} - x * \|_\infty}$ *with* $k < 1$,
then: $\limsup_{t \longrightarrow +\infty} \|e(t)\| \leq \lambda$

Bibliography

[1] N. Beniich, A. El Bouhtouri and D. Dochain *Input constrained adaptive local tracking for a nonlinear distributed parameter exothermic reaction models in tubular reactor* submitted to Automatica, 2006.

Timetable

Monday

Controller design for DPS		D. Dochain

Time	Title	Speaker
9.30–10.00	Welcome with coffee/tea	
10.00–10.15	Opening	J. Winkin
10.15–10.50	Volterra boundary control laws for 1-D parabolic nonlinear PDE's	M. Krstic
10.50–11.25	Robustness of stability of observers	L. Paunonen
11.25–12.00	An H_∞-observer at the boundary of an infinite-dimensional system	D. Vries
12.00–12.35	Predictive control of distributed parameter systems	P. Christofides
12.35–14.00	Lunch	
Linear systems theory		G. Weiss
14.00–14.35	Relation between the growth of $\exp(At)$ and $((A+I)(A-I)^{-1})^n$	N. Besseling
14.35–15.10	The observer infinite-dimensional Sylvester equation	Z. Emirsajlow
15.10–15.40	Coffe/Tea break	
15.40–16.15	Spectral properties of pseudo-resolvents under structured perturbations	B. Jacob
16.15–16.50	On the Carleson measure criterion in linear systems theory	B. Haak
16.50–17.25	Diffusive representation for fractional Laplacian and other non-causal pseudo-differential operators	D. Matignon

Tuesday

Control of systems described by p.d.e.'s		E. Jonckheere
Time	**Title**	**Speaker**
9.00–9.35	Motion planning of reaction-diffusion system arising in combustion and electrophysiology	C. Prieur
9.35–10.10	Control design of a distributed parameter fixed-bed reactor	I. Aksikas
10.10–10.40	Coffee/Tea break	
10.40–11.15	Scheduling of sensor network for detection of moving intruder	M. Demetriou
11.15–11.50	Switched Pritchard-Salamon systems with applications to moving actuators	O. Iftime
11.50–14.00	Lunch	
Control of DPS: A tribute to Frank M. Callier		R. Curtain
14.00–14.35	The motion planning problem and exponential stabilization of a heavy chain	P. Grabowski
14.35–15.10	A historical journey through the internal stabilization problem	A. Quadrat
15.10–15.40	Coffee/Tea break	
15.40–16.15	Approximate tracking for stable infinite-dimensional systems using sampled-data tuning regulators	H. Logemann
16.15–16.50	Problems of robust regulation in infinite-dimensional spaces	S. Pohjolainen
16.50–17.05	A tribute to Frank M. Callier	J. Winkin
17.05–19.00	Belgian beer and cheese party	

Wednesday

Neutral systems		F. Callier
Time	**Title**	**Speaker**
9.00–9.35	Stabilization of fractional delay systems of neutral type with single delay	C. Bonnet
9.35–10.10	Stability and computation of roots in delayed systems of neutral type	M. Peet
10.10–10.40	Coffee/Tea break	
10.40–11.15	What can regular linear systems do for neutral equations?	S. Hadd
11.15–11.50	On controllabilty and stabilizability of linear neutral type systems	R. Rabah
11.50–12.25	Coprime factorization for irrational functions	M. Opmeer
12.25–14.00	Lunch	
	Free/Hike	

Thursday

Energy methods		M. Tucsnak
Time	**Speaker**	**Title**
9.00–9.35	A class of passive time-varying well-posed linear systems	R. Schnaubelt
9.35–10.10	Lyapunov control of a particle in a finite quantum potential well	M. Mirrahimi
10.10–10.40	Coffee/Tea break	
10.40–11.15	Past, future, and full behaviors of passive state/signal systems	O. Staffans
11.15–11.50	Strong Stabilization of almost passive systems	R. Curtain
11.50–14.00	Lunch	
Controllability, observability, stabilizability, well-posedness		O. Staffans
14.00–14.35	Lure feedback systems with both unbounded control and observation: well-posedness and stability using nonlinear semigroups	F. Callier
14.35–15.10	A sharp geometric condition for the exponential stabilizability of a square plate by moment feedbacks only	K. Ammari
15.10–15.40	Coffe/Tea break	
15.40–16.15	Fast and strongly localized observation for the Schrödinger equation	M. Tucsnak
16.15–16.50	Exact controllability of Schrödinger type systems	G. Weiss
16.50–17.25	Controllability of the nonlinear Korteweg-de Vries equation for critical spatial lengths	E. Crépeau
19.00–24.00	Conference dinner	

Friday

Time	Title	Speaker
	Properties of linear systems	B. Jacob
Time	**Title**	**Speaker**
9.00–9.35	Well-posedness and regularity of hyperbolic systems	H. Zwart
9.35–10.10	Casimir functions and interconnection of boundary port Hamiltonian systems	Y. Le Gorrec
10.10–10.40	Coffee/Tea break	
10.40–11.15	Compactness of the difference between two thermoelastic semigroups	L. Maniar
11.15–11.50	On nonexistence of maximal asymptotics for certain linear equations in Banach space	G. Sklyar
11.50–13.30	Lunch	
	Non-linear p.d.e.'s, theory and applications	M. Demetriou
13.30–14.05	A biologically inspired synchronization of lumped parameter oscillators through a distributed parameter channel	E. Jonckheere
14.05–14.40	Boundary control of a channel in presence of small perturbations: a Riemann approach	V. Dos Santos
15.40–15.00	Coffee/Tea break	
15.00–15.35	Boundary control of a channel: internal model boundary control	Y. Touré
15.35–16.10	Constrained adaptive control for a nonlinear distributed parameter tubular reactor	D. Dochain
16.10–16.30	Farewell	

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